U.S. Coast and Geodetic Survey
Cartography

Serial No. 77

DEPARTMENT OF COMMERCE

U. S. COAST AND GEODETIC SURVEY

E. LESTER JONES, SUPERINTENDENT

CARTOGRAPHY

THE LAMBERT CONFORMAL CONIC PROJECTION

WITH TWO STANDARD PARALLELS

INCLUDING

A Comparison of the Lambert Projection with the Bonne and Polyconic Projections

BY

CHARLES H. DEETZ

Cartographer, United States Coast and Geodetic Survey

Special Publication No. 47

PRICE, 75 CENTS

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PREFACE.

By reason of an increasing demand for information on Lambert's projection and its relation to other well-known projections, the author of this paper is striving not only to present the salient features of the projection in question but also to elucidate the original formulas in such a manner as to convey to the cartographer and the public a thorough appreciation of the properties involved and their application to chart construction.

Very little is found in textbooks on the subject of Lambert's projection—seldom more than a paragraph and rarely more than a page. Illustrations for a thorough understanding of projections or their construction are by many authors deemed unnecessary, and scant formulas with no connecting link toward their direct application is the usual method. Some of the best projections have remained in obscurity for a century or more because they are not understood.

The approximate formula for the Lambert projection, as it is employed in France, is given first in detail, and the plates at the end of the book are in illustration of this method.

The rigid Lambert formula as presented by Gauss, by whose name the projection is sometimes known, is given next in detail also.

The writer believes the latter formula should prevail at all times. It is by this rigid system that the projection becomes exactly conformal, or, as stated by Gauss, "the model and the picture are made conformal in their minutest parts."

Following the rigid Lambert formula, a demonstration of its application to a map of the United States is given, this subject being further discussed in Part II.

Some repetition has been necessary in this paper. This is largely in essentials and with the view to impress upon the reader the true value of this projection, as well as to caution him against any haphazard selection of a projection to meet the requirements of any particular mapping problem.

Anyone wishing data to enable him to construct a Lambert projection should consult the section commencing with "Construction of a Lambert Conformal Conic Projection with Two Standard Parallels," page 17. The necessary tables follow this section and the plate explanatory of the construction is No. III at the end of this manual.

Special tables for constructing a Lambert projection in the region of the French war zone are published separately, as a supplement to this manual; also certain essential conversion tables. All the elements were calculated by the First Army of France for their general war map, with the origin of coordinates at the intersection of latitude 55 grades north, and longitude 6 grades east of Paris.

The author takes pleasure in acknowledging valuable assistance rendered in the preparation of this paper by Messrs. Oscar S. Adams and Walter D. Sutcliffe, geodetic computers, and Mr. Harlow Bacon, cartographer, United States Coast and Geodetic Survey.

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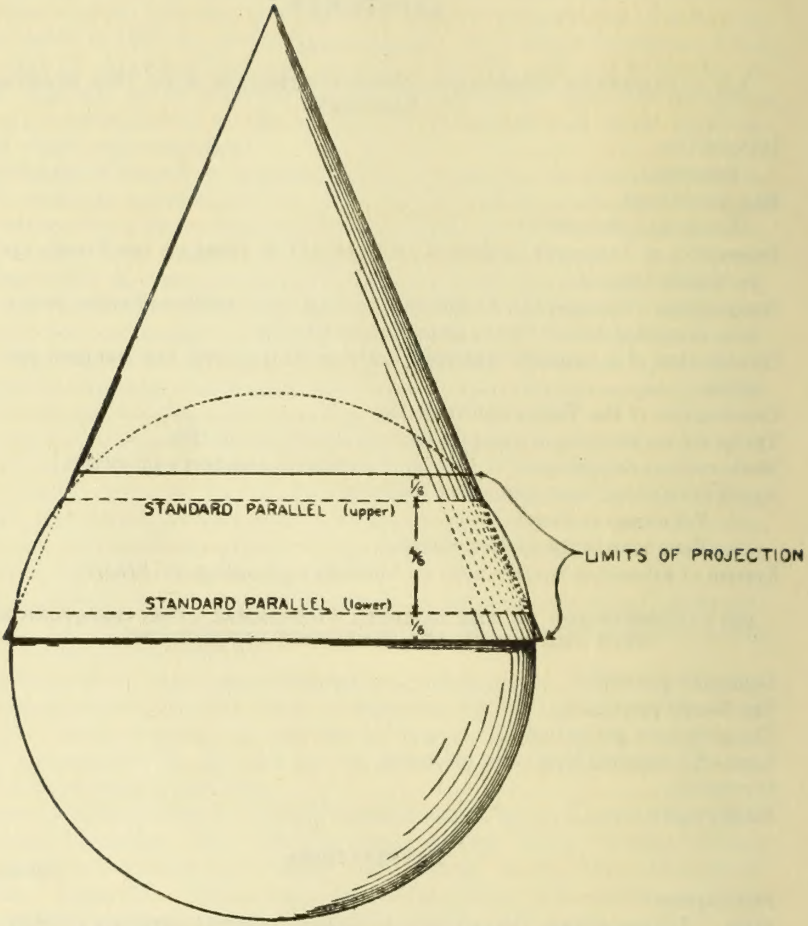

LAMBERT'S CONFORMAL CONIC PROJECTION
Diagram illustrating the intersection of a cone and sphere along the two standard parallels.

FRONTISPIECE.

THE LAMBERT CONFORMAL CONIC PROJECTION WITH TWO STANDARD PARALLELS, INCLUDING A COMPARISON OF THE LAMBERT PROJECTION WITH THE BONNE AND POLYCONIC PROJECTIONS.

Part 1.—LAMBERT'S CONFORMAL CONIC PROJECTION WITH TWO STANDARD PARALLELS.

INTRODUCTION.

In order to meet the need for a system of map projection in which a combination of minimum angular and scale distortion may be obtained, the French have adopted as a basis for new battle maps the one known as "Lambert's conformal conic projection."

For a base map covering a zone of 500 kilometers in width, or 250 kilometers on either side of the central parallel 49° 30′ (= 55 grades), this projection shows a degree of precision which is unique and answers every requirement for knowledge of orientation, distances, and quadrillage (system of kilometric squares). It is admirably adapted to a region of predominating east and west dimensions, and with it all the northeastern region of France, as well as Belgium and part of Germany, can be represented on one map. It can be extended east and west as far as desired, the projection remaining conformal throughout.

In this projection the angular distortion is exceedingly small, and linear distortion throughout the map no more than 0.05 per cent, which may be considered as practically negligible.

It is this projection—Lambert's conformal conic—that becomes the subject of this paper.

HISTORICAL.

Lambert, Johann Heinrich (1728–1777), physicist, mathematician, and astronomer, was born at Mülhausen, Alsace. He was of humble origin and it was entirely due to his own efforts that he obtained his education. In 1764, after some years in travel, he removed to Berlin, where he received many favors at the hand of Frederick the Great, and was elected a member of the Royal Academy of Sciences of Berlin, and in 1774 edited the Ephemeris.

He had the facility for applying mathematics to practical questions. The introduction of hyperbolic functions to trigonometry was due to him, and his discoveries in geometry are of great value, as well as his investigations in physics and astronomy. He was also the author of several remarkable theorems on conics which bear his name.

We are indebted to A. Wangerin, in Ostwald's Klassiker, 1894, for the following tribute to Lambert's contribution to cartography:

> The importance of Lambert's work consists mainly in the fact that he was the first to make general investigations upon the subject of map projection. His predecessors limited themselves to the investigations of a single method of projection, especially the perspective, but Lambert considered the problem of the representation of a sphere upon a plane from a higher standpoint and stated certain general conditions that the representation was to fulfill, the most important of these being the preservation of angles or conformality, and equal surface or equivalence. These two properties, of course, can not be attained in the same projection.
>
> Although Lambert has not fully developed the theory of these two methods of representation, yet he was the first to express clearly the ideas regarding them. The former—conformality—has become of the greatest importance to pure mathematics as well as the natural sciences, but both of them are of great significance to the cartographer. It is no more than just, therefore, to date the beginning of a new epoch in the science of map projection from the appearance of Lambert's work. Not only is his work of importance for the generality of his ideas but he has also succeeded remarkably well in the results that he has attained.

The manner in which Lambert attacks and solves his problems is very instructive. He has developed several methods of projection that are not only interesting but are to-day in use among cartographers, the most important of these being the subject matter of this treatise, the "Conformal conic projection," which appeared in his Beiträge zum Gebrauche der Mathematik und deren Anwendung, volume 3, Berlin, 1772.

Among other projections devised by Lambert, besides the one here discussed, and one having unusual merit, is his "Azimuthal equivalent projection," which is briefly described at the end of this paper.

MAP PROJECTIONS.

In the construction of maps the initial problem is the representation of a portion or all of the curved surface of the earth on a plane.

As a curved (or spheroidal) surface can not be fitted to a plane without distortion, such a representation must necessarily involve a certain amount of approximation or compensation.

The object, then, is to devise some system of projection best adapted to meet the requirements the map is to fulfill, whether the desirable conditions be a matter of correct angles between meridian and parallel, scaling properties, equivalence of areas, rhumb lines, etc.

Some of the elements desired may be retained at the expense of others, or a compromise may be adopted. A projection for an area of predominating east and west dimensions would not be suited for an area of predominating north and south dimensions. Thus, a map of the United States with its wide longitude and comparatively narrow latitude should never be drawn on a polyconic projection, as appears to be the case in some of our Government bureaus. The linear meridional distortion of such a projection is as much as $6\frac{1}{2}$ per cent on the Pacific coast. By using Lambert's conformal conic projection the maximum linear distortion in a map of the United States can be reduced to 1 per cent.

The use of a projection for a purpose to which it is not best suited is therefore generally unnecessary and should be avoided.

All these projections have certain unquestionable merits as well as equally serious defects, and each region to be mapped should be made the subject of special study and, as a rule, that system of projection adopted which will give the best results for the area under consideration.

CONFORMAL.

A conformal projection or development takes its name from the property that all small or elementary figures found or drawn upon the surface of the earth retain their original forms upon the projection.

This implies that—

All angles between intersecting lines or curves are preserved;

For any given point (or restricted locality) the ratio of the length of a linear element on the earth's surface to the length of the corresponding map element is constant for all azimuths or directions in which the element may be taken.

Arthur R. Hinks, M. A., in his treaties on "Map projections," defines *orthomorphic*, which is another term for *conformal*, as follows:

> If at any point the scale along the meridian and the parallel is the same (not correct, but the same in the two directions) and the parallels and meridians of the map are at right angles to one another, then the shape of any very small area on the map is the same as the shape of the corresponding small area upon the earth. The projection is then called orthomorphic (right shape).

DESCRIPTION OF LAMBERT'S CONFORMAL CONIC PROJECTION, BASED ON THE FRENCH APPROXIMATE FORMULA.*

The meridian of O on the sphere is represented by a straight line co on the projection, and the parallel OQ by a circle the center of which is at c on the line co, and the radius of which is

$$co = CQ' = N_0 \cot L_0 \quad \text{(See fig. 5 and key to abbreviations.)}$$

The length a of the arcs of parallels on the initial parallel is laid off upon the circle of radius co.

The meridians are represented by straight lines radiating from c at intervals proportional to the longitude, the angle between any meridian and the initial meridian being $(M - M_0) \sin L_0$.

ra and oq (fig. 3) are arcs of the parallels corresponding to the difference of longitude indicated by the angle at c.

The other parallels are also represented by circles with center at c, spaced in such a way as to make the projection conformal; that is, to preserve angles unchanged.†

It can be proved that in order to realize this condition the spacing of the parallels should be

$$\beta + \frac{\beta^3}{6\rho_0^2} \quad \text{(See key to abbreviations.)}$$

This, then, will give us a conformal projection on a tangent cone; that is, a projection with one standard parallel. The scale will be correct along this initial parallel; beyond that the scale will be increasingly large. Each point on the map has a scale characteristic of that point, and meridians and parallels intersect at right angles. But by reducing the scale of the projection by a constant ratio m (nearly unity; see key to abbreviations), the lengths at and near the initial parallel are diminished, and we pass to a conformal conic projection with two standard parallels instead of one (fig. 2), embodying all the properties involved in the definition of the term "Conformal."

The standard parallels are usually chosen at one-sixth and five-sixths of the total length of that portion of the central meridian to be represented. The scale between these standard parallels is a little too small, and beyond them too large.

---

* Translated from the French, with additions, and illustrated by figures 1 to 4.

† This is rigorously true only within certain distances from the point of origin if the approximate formula given here is adhered to. If the less simple but rigorous formula be used, the projection is exactly conformal.

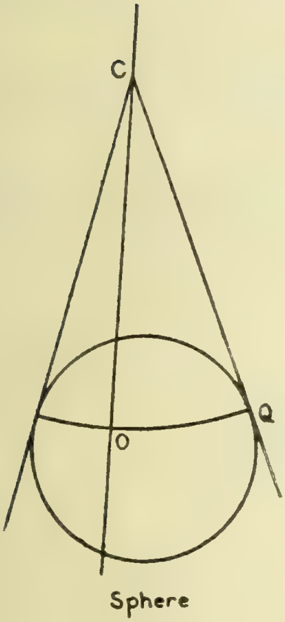

Fig. 1.

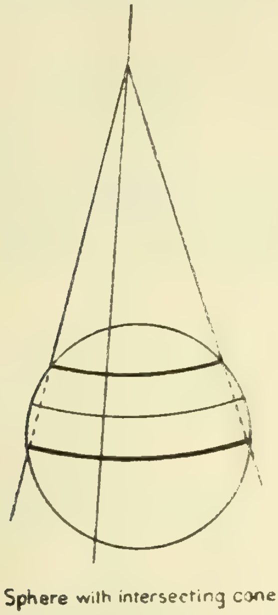

Fig. 2.

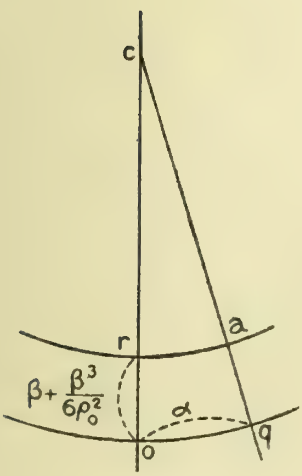

Fig. 3.

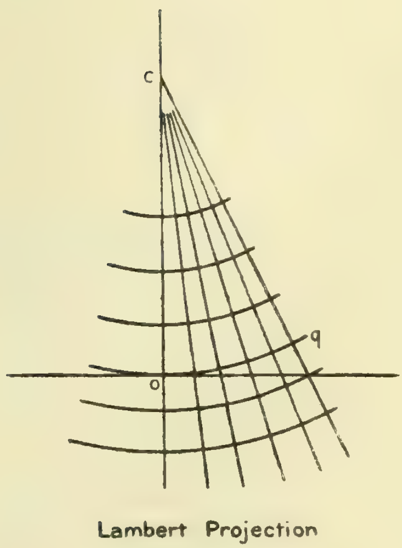

Fig. 4.

COMPUTATION OF GEOGRAPHIC COORDINATES FOR LAMBERT'S CONFORMAL CONIC PROJECTION, ACCORDING TO THE FRENCH APPROXIMATE FORMULA.

LOCATION OF POINTS ON THE EARTH'S SURFACE BY GEOGRAPHIC COORDINATES.

KEY TO ABBREVIATIONS.

Let O be a point whose geographic coordinates are L_o M_o, located at the center of the area to be represented and taken as the origin of the projection.

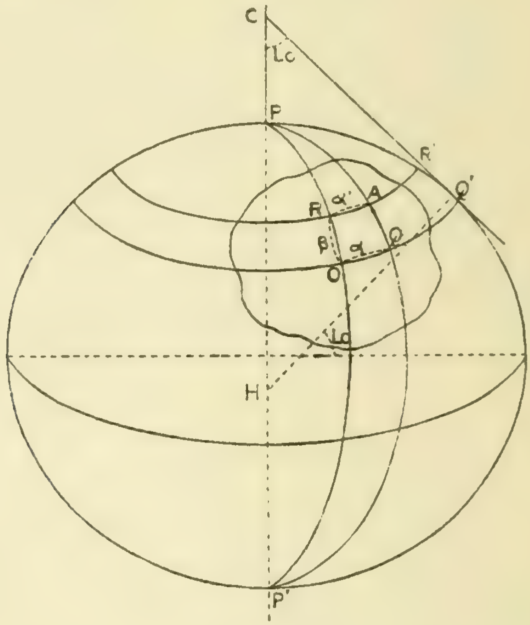

FIG. 5.

POP' = the meridian passing through O, whose longitude is M_o; and OQQ' the parallel of latitude L_o.

A = any point whose coordinates are L and M, located on the meridian PAP' and on the parallel RAR'.

α = the length of the parallel of O included between the two meridians.

α' = the length of the arc of the parallel of A included between the two meridians.

β = the length of the arc of the meridian included between parallels, reckoned from O.

$N_o = HQ'$ = the length of the normal to the surface at the parallel of O, produced to the minor axis.

= radius of curvature in the prime vertical.

R_o = radius of curvatue in the meridian.

ρ_o = mean radius of curvature of the ellipsoid at the origin O.

$= \sqrt{R_o\ N_o}$.

$r_o = co$ = radius of circle representing middle parallel (fig. 4).

θ = the angle on the projection between the meridian of any point A and the initial meridian = convergence of meridians.

m = constant ratio (nearly unity) by which the scale of projection is reduced. This diminishes the scale along the central parallel, preserves the scale along two selected parallels equidistant from the center, and increases it beyond these parallels.

$m = 1 - \frac{1}{2037}$ is the ratio adopted at the French front, lengths being preserved on the two parallels of 53 grades ($=47°$ $42'$) and 57 grades ($=51°$ $18'$), situated two grades on either side of the initial parallel.

Numerical term $(2037) = \frac{2\rho_o^2}{\beta^2}$ where β = length of meridian from central parallel to one of the two selected parallels on which the lengths are preserved.

This reduction factor is approximately the one used by the French and determined in the following manner:

$$\Delta r = \beta + \frac{\beta^3}{6\rho_o^2}$$

$$\frac{d(\Delta r)}{d\beta} = 1 + \frac{\beta^2}{2\rho_o^2}$$

We can determine a reduction factor that will make the value $\frac{d(\Delta r)}{d\beta}$ equal to unity, and in this way hold the scale true upon any two selected parallels at equal distances north and south of the middle parallel.

We must determine x so that

$$\left(1 - \frac{1}{x}\right)\left(1 + \frac{\beta^2}{2\rho_o^2}\right) = 1$$

$$\text{or } 1 - \frac{1}{x} = \frac{1}{1 + \frac{\beta^2}{2\rho_o^2}}$$

or approximately,

$$1 - \frac{1}{x} = 1 - \frac{\beta^2}{2\rho_o^2}$$

$$\text{hence } \frac{1}{x} = \frac{\beta^2}{2\rho_o^2}$$

$$x = \frac{2\rho_o^2}{\beta^2}$$

$$^*\text{Normal } N_0=\frac{a}{(1-e^2\sin^2\phi)^{\frac{1}{2}}} \text{ or } \frac{1}{N_0}=\frac{(1-e^2\sin^2\phi)^{\frac{1}{2}}}{a}$$

$$^*\text{Factor } A=\frac{(1-e^2\sin^2\phi)^{\frac{1}{2}}}{a\sin 1''}=\frac{1}{N_0\sin 1''}$$

Then $N_0=\frac{1}{A\sin 1''}$,

colog $N_0=\log A + \log\sin 1''$

*log A for 49° 30′ (middle parallel)	$=8.5088750-10$
log sin 1″	$=4.6855749-10$
colog N_0	$=3.1944499-10$

$\therefore \log N_0=6.8055501$ (I)

$$^*R_0=\frac{a(1-e^2)}{(1-e^2\sin^2\phi)^{\frac{3}{2}}}$$

$$^*\text{Factor } B=\frac{(1-e^2\sin^2\phi)^{\frac{3}{2}}}{a(1-e^2)\sin 1''}$$

$$\therefore R_0=\frac{1}{B\sin 1''}$$

colog $R_0=\log B+\log\sin 1''$

*log B for 49° 30′	$=8.5101216-10$
log sin 1″	$=4.6855749-10$
colog R_0	$=3.1956965-10$

$\therefore \log R_0=6.8043035$ (II)

$r_0=N_0\cot L_0=N_0\cot 49° 30'$

log N_0	$=6.8055501$ (from I)
log cot 49° 30′	$=9.9314989$
	6.7370490

$\therefore r_0=5{,}458{,}195$ = radius for middle parallel 49° 30′ (III)

In order to realize the condition that the concentric circles be spaced in such a way as to make the projection conformal—that is, to preserve angles unchanged—the distance of any given parallel from the middle parallel should be equal to

$$\beta+\frac{\beta^3}{6\rho_0^2}\dagger$$

β being the arc of the meridian reckoned from O, and ρ_0 the mean radius of curvature of the ellipsoid at the origin O.

*From Appendix 9, Coast and Geodetic Survey Report, 1894.

†This is a development of Δr in a Taylor series to the third power.

The Development by Taylor's Theorem of the function

$$f(p)=r=K\tan^l\frac{p}{2}\cdot\left(\frac{1+\epsilon\cos p}{1-\epsilon\cos p}\right)^{\frac{l\epsilon}{2}}$$

in terms of the arc (β) along the meridian for the case of a cone tangent at p_0, the scale being held exact upon the parallel of tangency, follows:

$$f(p)=r=K\tan^l\frac{p}{2}\cdot\left(\frac{1+\epsilon\cos p}{1-\epsilon\cos p}\right)^{\frac{l\epsilon}{2}}.$$

By Taylor's theorem

$$f(p_0+\Delta p)=r_0+\Delta r=f(p_0)+f'(p_0)\beta+\frac{f''(p_0)}{\underline{|2}}\beta^2+\frac{f'''(p_0)}{\underline{|3}}\beta^3+\ldots+\frac{f^n(p_0)}{\underline{|n}}\beta^n+\ldots.$$

The primes denote differentiation with regard to the arc β, the relation between $d\beta$ and dp being

$$d\beta=\frac{a(1-\epsilon^2)dp}{(1-\epsilon^2\cos^2 p)^{\frac{3}{2}}}.$$

Taking logarithms the equation becomes

$$\log f(p)=\log K+l\log\tan\frac{p}{2}+\frac{l\epsilon}{2}\log(1+\epsilon\cos p)-\frac{l\epsilon}{2}\log(1-\epsilon\cos p).$$

By differentiation,

$$(a)\qquad \frac{f'(p)}{f(p)}=\left[\frac{l}{\sin p}-\frac{l\epsilon^2\sin p}{1+\epsilon\cos p}-\frac{l\epsilon^2\sin p}{1-\epsilon\cos p}\right]\cdot\frac{dp}{d\beta}$$

$$=\frac{l(1-\epsilon^2)}{(1-\epsilon^2\cos^2 p)\sin p}\cdot\frac{(1-\epsilon^2\cos^2 p)^{\frac{3}{2}}}{a(1-\epsilon^2)}=\frac{l(1-\epsilon^2\cos^2 p)^{\frac{1}{2}}}{a\sin p}.$$

Taking logarithms of equation (a), we have

$$\log f'(p)-\log f(p)=\log l+\tfrac{1}{2}\log(1-\epsilon^2\cos^2 p)-\log a-\log\sin p.$$

Differentiating again,

$$\frac{f''(p)}{f'(p)}-\frac{f'(p)}{f(p)}=\left(\frac{\epsilon^2\sin p\cos p}{1-\epsilon^2\cos^2 p}-\frac{\cos p}{\sin p}\right)\cdot\frac{dp}{d\beta}=\frac{-(1-\epsilon^2)\cos p}{(1-\epsilon^2\cos^2 p)\sin p}\cdot\frac{(1-\epsilon^2\cos^2 p)^{\frac{3}{2}}}{a(1-\epsilon^2)}$$

$$=-\frac{(1-\epsilon^2\cos^2 p)^{\frac{1}{2}}\cos p}{a\sin p}.$$

Substituting the value of $\frac{f'(p)}{f(p)}$ from (a), the equation becomes

$$(b)\qquad \frac{f''(p)}{f'(p)}=\frac{(l-\cos p)(1-\epsilon^2\cos^2 p)^{\frac{1}{2}}}{a\sin p}.$$

Differentiating equation (b), we get

$$(c)\qquad \frac{f'''(p)}{f'(p)}-\left[\frac{f''(p)}{f'(p)}\right]^2=\left[\frac{1-\epsilon^2\cos^2 p)^{\frac{1}{2}}}{a}+\frac{(l-\cos p)\epsilon^2\cos p}{a(1-\epsilon^2\cos^2 p)^{\frac{1}{2}}}\right.$$

$$\left.-\frac{(l-\cos p)(1-\epsilon^2\cos^2 p)^{\frac{1}{2}}\cos p}{a\sin^2 p}\right]\cdot\frac{dp}{d\beta}=\left[\frac{(1-\epsilon^2\cos^2 p)^{\frac{1}{2}}}{a}\right.$$

$$\left.-\frac{(1-\epsilon^2)(l-\cos p)\cos p}{a(1-\epsilon^2\cos^2 p)^{\frac{1}{2}}\sin^2 p}\right]\frac{(1-\epsilon^2\cos^2 p)^{\frac{3}{2}}}{a(1-\epsilon^2)}=\frac{(1-\epsilon^2\cos^2 p)^2}{a^2(1-\epsilon^2)}$$

$$-\frac{(l-\cos p)(1-\epsilon^2\cos^2 p)\cos p}{a^2\sin^2 p}.$$

These derivatives must now be evaluated for the co-latitude p_o. Since the cone is tangent at the co-latitude p_o, we have

$$f(p_o)=r_o=\frac{a \tan p_o}{(1-\epsilon^2 \cos^2 p_o)^{\frac{1}{2}}}.$$

Since the scale is to be held exact upon the parallel of tangency, we have the condition

$$f'(p_o)=\left(\frac{dr}{d\beta}\right)_0=1,$$

this being the general relation between a curve and its tangent.

From equation (a)

$$f'(p_o)=\frac{l(1-\epsilon^2 \cos^2 p_o)^{\frac{1}{2}}}{a \sin p_o} f(p_o)=1.$$

Substituting the above value of $f(p_o)$, this becomes

$$\frac{l(1-\epsilon^2 \cos^2 p_o)^{\frac{1}{2}}}{a \sin p_o} \cdot \frac{a \tan p_o}{(1-\epsilon^2 \cos^2 p_o)^{\frac{1}{2}}}=1,$$

or

$$l=\cos p_o.$$

By substituting this value of l in equation (b), we find that

$$f''(p_o)=o.$$

If these values of $l, f'(p_o)$, and $f''(p_o)$ are substituted in equation (c), there results

$$f'''(p_o)=\frac{(1-\epsilon^2 \cos p_o)^2}{a^2(1-\epsilon^2)}.$$

If ρ_0 is the geometric mean of the radii of curvature at the point p_o, we can express this equation in the form

$$f'''(p_o)=\frac{1}{\rho_0{}^2}$$

When these values are substituted in the Taylor's series, the series becomes

$$r_o+\Delta r=r_o+\beta+\frac{\beta^3}{6\rho_0{}^2}+\ldots;$$

$$\text{or, } \Delta r=\beta+\frac{\beta^3}{6\rho_0{}^2}+\ldots$$

This is the correct value of the series to the third power of β.

The following computations serve to illustrate the application of this expression:

β (meridional arc from 49° 00′ to 49° 30′ = 55607. 3 meters.
β (meridional arc from 49° 30′ to 50° 00′ = 55612. 2 meters.
$\rho_0=\sqrt{R_o N_o}$

$$\log R_o = 6.\ 8043035 \text{ (from II)}$$
$$\log N_o = 6.\ 8055501 \text{ (from I)}$$
$$2\overline{)13.\ 6098536}$$
$$\log \rho_0 = 6.\ 8049268$$

∴ spacing of the parallels in formula $\beta + \frac{\beta^3}{6\rho_o^2}$

becomes,

$$55607.3 + \frac{55607.3^3}{6\rho_o^2} = 55607.3 + 0.7 = 55608.0 = \text{spacing between parallels } 49^\circ\ 00' \text{ and } 49^\circ\ 30'$$

In like manner,

$$55612.2 + 0.7 = 55612.9 = \text{spacing between parallels } 49^\circ\ 30' \text{ and } 50^\circ\ 00'$$

$48^\circ\ 00'$ to $49^\circ\ 30' = 166807.2$ (= 55607.3 + 111199.9) + 19.0 = 166826.2
$49^\circ\ 30'$ to $51^\circ\ 00' = 166851.2$ (= 55612.2 + 111239.0) + 19.0 = 166870.2
$47^\circ\ 00'$ to $49^\circ\ 30' = 277987.4$ (= 166807.2 + 111180.2) + 87.9 = 278075.3
$49^\circ\ 30'$ to $52^\circ\ 00' = 278109.5$ (= 166851.2 + 111258.3) + 88.0 = 278197.5

Combining the above with (III), the radii become:

Radius for $47^\circ\ 00'$ = 5, 458, 195 + 278, 075 = 5, 736, 270 meters.
Radius for $48^\circ\ 00'$ = 5, 458, 195 + 166, 826 = 5, 625, 021 meters.
Radius for $49^\circ\ 00'$ = 5, 458, 195 + 55, 608 = 5, 513, 803 meters.
Radius for $50^\circ\ 00'$ = 5, 458, 195 − 55, 613 = 5, 402, 582 meters.
Radius for $51^\circ\ 00'$ = 5, 458, 195 − 166, 870 = 5, 291, 325 meters.
Radius for $52^\circ\ 00'$ = 5, 458, 195 − 278, 197 = 5, 179, 998 meters.

Reducing scale of projection by constant ratio

$$m = 1 - \frac{1}{2037}$$

r for $47^\circ\ 00'$ = 5, 733, 454 meters.
r for $48^\circ\ 00'$ = 5, 622, 260 meters.
r for $49^\circ\ 00'$ = 5, 511, 096 meters.
r for $49^\circ\ 30'$ = 5, 455, 515 meters.
r for $50^\circ\ 00'$ = 5, 399, 930 meters.
r for $51^\circ\ 00'$ = 5, 288, 727 meters.
r for $52^\circ\ 00'$ = 5, 177, 455 meters.

Spacing between parallels becomes:

$47^\circ\ 00'$ to $49^\circ\ 30'$ = 277, 939 meters.
$48^\circ\ 00'$ to $49^\circ\ 30'$ = 166, 745 meters.
$49^\circ\ 00'$ to $49^\circ\ 30'$ = 55, 581 meters. (IV)
$49^\circ\ 30'$ to $50^\circ\ 00'$ = 55, 585 meters.
$49^\circ\ 30'$ to $51^\circ\ 00'$ = 166, 788 meters.
$49^\circ\ 30'$ to $52^\circ\ 00'$ = 278, 060 meters.

52829°—18——2

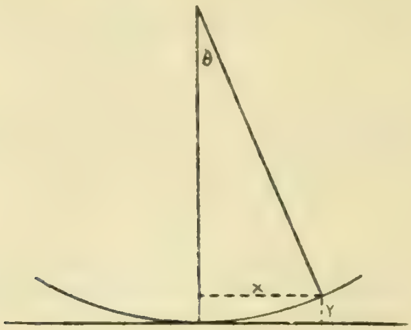

FIG. 6.

$$\theta = \text{convergence of meridians} = (M - M_o) \sin L_o$$
$$= (M - M_o) \sin 49^\circ\ 30'$$
$$= (M - M_o)\ (0.76)$$

$\therefore$ convergence of 1° long. from central meridian $= 0.76^\circ = 45'\ 36''$
convergence of 2° long. from central meridian $= 0.76 \times 2 = 1^\circ\ 31'\ 12''$
convergence of 3° long. from central meridian $= 0.76 \times 3 = 2^\circ\ 16'\ 48''$
convergence of 4° long. from central meridian $= 0.76 \times 4 = 3^\circ\ 02'\ 24''$
convergence of 5° long. from central meridian $= 0.76 \times 5 = 3^\circ\ 48'\ 00''$
convergence of 6° long. from central meridian $= 0.76 \times 6 = 4^\circ\ 33'\ 36''$
convergence of 7° long. from central meridian $= 0.76 \times 7 = 5^\circ\ 19'\ 12''$

$$x = r \sin \theta$$
$$y = r - r \cos \theta = r\ (1 - \cos \theta) = 2\ r \sin^2 \frac{\theta}{2}$$
$$\text{or, } y = x \tan \frac{\theta}{2}$$
$$\theta = 45'\ 36'' \text{ for } 1^\circ \text{ from central meridian.}$$

Applying the above, we have the following:

FOR LATITUDE 47° 00′, r=5,733,454.

	1° long.	2°	3°	4°	5°	6°	7°
x..........	76,049	152,085	228,094	304,063	379,978	455,827	531,595
y..........	504	2,017	4,539	8,068	12,605	18,149	24,698
FOR LATITUDE 52° 00′, r=5,177,455.							
x..........	68,674	137,337	205,975	274,577	343,130	411,623	480,044
y..........	455	1,822	4,099	7,286	11,383	16,389	22,302

(V)

CONSTRUCTION OF A LAMBERT CONFORMAL CONIC PROJECTION WITH TWO STANDARD PARALLELS.

[See Plate III.]

The coordinates for the projection of northeastern France and Germany (Plate I), given in the preceding computations, were determined from the French approximate formula for intervals of 1°.

The two standard parallels chosen by the French for this map are 53 and 57 grades (1 grade = 1/100 of a quadrant in the French notation). The middle parallel is 55 grades. The equivalents in our notation are:

$$53^G = 47° 42'$$
$$55^G = 49° 30'$$
$$57^G = 51° 18'$$

The notation followed on this plate is in degrees and not in grades.

Draw a straight line ***ab*** (Plate III) for a central meridian and a construction line ***cd*** perpendicular to it, each line to be as central to the sheet as the selected interval of latitude and longitude will permit.

To insure greater accuracy on large sheets, the longer line of the two should be drawn first, and the shorter line erected perpendicular to it.

Let ***cd*** represent the construction line for the middle parallel 49° 30′. From the preceding computations (see IV, p. 15) lay off on the central meridian the distances representing the spacing between the middle parallel and the other parallels to be represented on the map. For instance, the distance between parallels 47° 00′ and 49° 30′ is 277,939 meters. Through the upper and lower points thus found, draw construction lines ***ef*** and ***gh*** parallel to ***cd***.

On these upper and lower construction lines ***ef*** and ***gh*** now lay off the x and y coordinates as given in the preceding computations. (See V, p. 16.)

For example, in longitude 10° (=6° from center of map, Plate III) lay off to the east on the lower construction line ***gh*** the value x (under 6°) =455,827 meters; and to the north the value y (under 6°)=18,149 meters.

At the upper end of y (indicated by a small circle on Plate III) we then have established the intersection of parallel 47° with the meridian 10°.

In like manner the x and y coordinates will establish the intersection of parallel 52° and the meridian 10°.

By drawing a straight line through the two points thus determined the meridian 10° is located on the map.

Apply the x and y coordinates along the upper and lower construction lines for the remaining meridians, which can then be drawn in.

To establish the parallels, connect the y points to right and left of the central meridian along the upper and lower construction lines, and we then have parallels 47° and 52° located. The y points are generally close enough together so that the chords thus drawn approximate the circle.

The remaining parallels can then be determined by subdividing all the meridional arcs between 47° and 52° not equally but in the same proportion as they appear along the central meridian.

The projection being completed, all meridians should be straight lines and all parallels of latitude arcs of concentric circles.

CONSTRUCTION OF THE TABLES AND THEIR USE.

CONSTRUCTION OF THE TABLES.

The parallel of 55^G was adopted as the central parallel and the spacings of the other parallels were computed by the formula

$$\Delta r = \beta + \frac{\beta^3}{6\rho_0^2}$$

The adoption of 0.76 as an approximate value for sin 55^G diminished the constant of the cone and consequently shortened the parallels. The radius for 55^G was determined so as to hold exactly the parallel of 53^G. The spacings of the parallels were then reduced 1 part in 2033, so that the scale along the meridian should be correct at 53^G. This rendered the scale exactly correct at 53^G, both along the parallel and along the meridian. As a result, at any given point of the map, the scale is the same in all directions, and hence the conformal quality is preserved. The other parallel that is held in scale is approximately 57^G.

The computations are based upon the Clarke ellipsoid of 1866. The meridional arcs were taken from the Polyconic Projection Tables, United States Coast and Geodetic Survey, Special Publication, No. 5.

USE OF TABLES FOR MAP PROJECTIONS IN NORTHEASTERN FRANCE.

[See end of preface in regard to special tables published separately.]

In Table II, under column "Spacing of parallels," we have given the distance in meters for each parallel from the middle parallel of the map of France; that is, the parallel of 55 grades.

By subtracting the values opposite the parallels to be represented on any projection, we obtain the spacings in meters between these same parallels.

In constructing a projection, however, it is better procedure to select the middle parallel to be mapped, and find the differences in meters between this middle parallel and all the other parallels in the same way.

In Table III (x's and y's), the first 10 lines give intervals of 0.02 grade and are intended for large scale projections only. The corresponding spacings of the parallels for each 0.02 grade can be obtained by division of the spacing of 0.1 grade intervals into five parts.

TABLE I.

M	θ (0.76 M)			Log sin θ	Log $\left(2\sin^2\frac{\theta}{2}\right)$
Grades	°	′	″		
0.02			49.248	6.3779635	2.4548970
0.04		1	38.496	6.6789934	3.0569570
0.06		2	27.744	6.8550847	3.4091396
0.08		3	16.992	6.9800234	3.6590168
0.10		4	06.240	7.0769334	3.8528368
0.12		4	55.488	7.1561146	4.0111994
0.14		5	44.736	7.2230614	4.1450930
0.16		6	33.984	7.2810532	4.2610768
0.18		7	23.232	7.3322056	4.3633820
0.2		8	12.480	7.3779630	4.4548968
0.3		12	18.72	7.5549538	4.8070792
0.4		16	24.96	7.6789918	5.0569560
0.5		20	31.20	7.7759009	5.2507760
0.6		24	37.44	7.8550510	5.4091376
0.7		28	43.68	7.9220265	5.5430304
0.8		32	49.92	7.9890169	5.6590136
0.9		36	56.16	8.0311677	5.7613180
1.0		41	02.40	8.0769232	5.8528318
1.1		45	08.64	8.1183137	5.9356160
1.2		49	14.88	8.1561000	6.0111920
1.3		53	21.12	8.1908593	6.0807150
1.4		57	27.36	8.2230413	6.1450830
1.5	1	01	33.60	8.2530015	6.2050078
1.6	1	05	39.84	8.2810271	6.2610638
1.7	1	09	46.08	8.3073525	6.3137198
1.8	1	13	52.32	8.3321726	6.3633654
1.9	1	17	58.56	8.3556498	6.4103256
2.0	1	22	04.80	8.3779222	6.4548764
2.1	1	26	11.04	8.3991073	6.4972528
2.2	1	30	17.28	8.4193062	6.5376574
2.3	1	34	23.52	8.4386067	6.5762652
2.4	1	38	29.76	8.4570852	6.6132300
2.5	1	42	36.00	8.4748090	6.6486846
2.6	1	46	42.24	8.4918371	6.6827486
2.7	1	50	48.48	8.5082220	6.7155268
2.8	1	54	54.72	8.5240107	6.7471126
2.9	1	59	00.96	8.5392447	6.7775896
3.0	2	03	07.20	8.5539619	6.8070330
3.1	2	07	13.44	8.5681960	6.8355106
3.2	2	11	19.68	8.5819778	6.8630842
3.3	2	15	25.92	8.5953351	6.8898086
3.4	2	19	32.16	8.6082931	6.9157350
3.5	2	23	38.40	8.6208750	6.9409100
3.6	2	27	44.64	8.6331023	6.9653752
3.7	2	31	50.88	8.6449940	6.9891696
3.8	2	35	57.12	8.6565680	7.0123296
3.9	2	40	03.36	8.6678412	7.0348578
4.0	2	44	09.60	8.6788284	7.0568744
4.1	2	48	15.84	8.6895439	7.0783178
4.2	2	52	22.08	8.7000008	7.0992446
4.3	2	56	28.32	8.7102112	7.1196784
4.4	3	00	34.56	8.7201865	7.1396424
4.5	3	04	40.80	8.7299371	7.1591574

TABLE II.

Latitude L	Spacing of parallels	Radius r	Log r
Grades.			
52.5	250 135.2	5 708 697.5	6.7565370
52.6	240 126.9	5 698 689.2	6.7557750
52.7	230 119.1	5 688 681.4	6.7550116
52.8	220 111.5	5 678 673.8	6.7542469
52.9	210 104.4	5 668 666.7	6.7534809
53.0	200 097.7	5 658 660.0	6.7527136
53.1	190 091.4	5 648 653.7	6.7519449
53.2	180 085.2	5 638 647.5	6.7511750
53.3	170 079.4	5 628 641.7	6.7504036
53.4	160 073.7	5 618 636.0	6.7496309
53.5	150 068.5	5 608 630.8	6.7488569
53.6	140 063.4	5 598 625.7	6.7480814
53.7	130 058.3	5 588 620.6	6.7473046
53.8	120 053.6	5 578 615.9	6.7465264
53.9	110 049.0	5 568 611.3	6.7457469
54.0	100 044.5	5 558 606.8	6.7449659
54.1	90 040.1	5 548 602.4	6.7441836
54.2	80 035.7	5 538 598.0	6.7433998
54.3	70 031.3	5 528 593.6	6.7426147
54.4	60 027.2	5 518 589.5	6.7418281
54.5	50 022.7	5 508 585.0	6.7410400
54.6	40 018.3	5 498 580.6	6.7402506
54.7	30 013.8	5 488 576.1	6.7394597
54.8	20 009.4	5 478 571.7	6.7386674
54.9	10 004.8	5 468 567.1	6.7378736
55.0	0.0	5 458 562.3	6.7370783
55.1	10 005.0	5 448 557.3	6.7362815
55.2	20 010.0	5 438 552.3	6.7354833
55.3	30 015.2	5 428 547.1	6.7346836
55.4	40 020.8	5 418 541.5	6.7338824
55.5	50 026.7	5 408 535.6	6.7330797
55.6	60 032.9	5 398 529.4	6.7322755
55.7	70 039.2	5 388 523.1	6.7314697
55.8	80 045.8	5 378 516.5	6.7306625
55.9	90 053.0	5 368 509.3	6.7298537
56.0	100 060.4	5 358 501.9	6.7290433
56.1	110 068.2	5 348 494.1	6.7282315
56.2	120 076.4	5 338 485.9	6.7274180
56.3	130 085.2	5 328 477.1	6.7266031
56.4	140 094.4	5 318 467.9	6.7257866
56.5	150 104.1	5 308 458.2	6.7249684
56.6	160 114.3	5 298 448.0	6.7241486
56.7	170 125.2	5 288 437.1	6.7233273
56.8	180 136.5	5 278 425.8	6.7225044
56.9	190 148.5	5 268 413.8	6.7216798
57.0	200 161.1	5 258 401.2	6.7208537
57.1	210 174.3	5 248 388.0	6.7200259
57.2	220 188.1	5 238 374.2	6.7191966
57.3	230 202.8	5 228 359.5	6.7183654
57.4	240 218.1	5 218 344.2	6.7175327
57.5	250 234.2	5 208 328.1	6.7166983
57.6	260 250.9	5 198 311.4	6.7158622
57.7	270 268.5	5 188 293.8	6.7150246
57.8	280 287.2	5 178 275.1	6.7141851
57.9	290 306.5	5 168 255.8	6.7133440
58.0	300 326.6	5 158 235.7	6.7125012

TABLE III

Longitude	Latitude									
	52.5ᴳ		52.6ᴳ		52.7ᴳ		52.8ᴳ		52.9ᴳ	
	x	y	x	y	x	y	x	y	x	y
Grades	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*
0.02...	1363.0	0.2	1360.6	0.2	1358.2	0.2	1355.8	0.2	1353.5	0.2
0.04...	2726.0	0.7	2721.2	0.6	2716.5	0.6	2711.7	0.6	2706.9	0.6
0.06...	4089.0	1.5	4081.9	1.5	4074.7	1.5	4067.5	1.5	4060.4	1.5
0.08...	5452.1	2.6	5442.5	2.6	5432.9	2.6	5423.4	2.6	5413.8	2.6
0.10...	6815.1	4.1	6803.1	4.1	6791.2	4.1	6779.2	4.0	6767.3	4.0
0.12...	8178.1	5.9	8163.7	5.8	8149.4	5.8	8135.1	5.8	8120.7	5.8
0.14...	9541.1	8.0	9524.4	8.0	9507.6	7.9	9490.9	7.9	9474.2	7.9
0.16...	10 904.1	10.4	10 885.0	10.4	10 865.9	10.4	10 846.8	10.4	10 827.7	10.3
0.18...	12 267.1	13.2	12 245.6	13.2	12 224.1	13.1	12 202.6	13.1	12 181.1	13.1
0.2....	13 630.1	16.3	13 606.2	16.2	13 582.3	16.2	13 558.4	16.2	13 534.6	16.2
0.3....	20 445.2	36.6	20 409.3	36.5	20 373.5	36.5	20 337.7	36.4	20 301.8	36.3
0.4....	27 260.2	65.1	27 212.4	65.0	27 164.6	64.9	27 116.8	64.7	27 069.0	64.6
0.5....	34 075.2	101.7	34 015.4	101.5	33 955.7	101.3	33 896.0	101.2	33 836.2	101.0
0.6....	40 890.1	146.4	40 818.4	146.2	40 746.7	145.9	40 675.1	145.7	40 603.4	145.4
0.7....	47 705.0	199.3	47 621.3	199.0	47 537.7	198.6	47 454.1	198.3	47 370 5	197.9
0.8....	54 519.8	260.3	54 424.2	259.9	54 328.6	259.4	54 233.0	259.0	54 137.5	258.5
0.9....	61 334.5	329.5	61 227.0	328.9	61 119.4	328.3	61 011.9	327.8	60 904.4	327.2
1.0....	68 149.1	406.8	68 029.7	406.1	67 910.2	405.4	67 790.7	404.6	67 671.3	403.9
1.1....	74 963.6	492.2	74 832.2	491.4	74 700.8	490.5	74 569.4	489.6	74 438.0	488.8
1.2....	81 778.1	585.8	81 634.7	584.7	81 491.4	583.7	81 348.0	582.7	81 204.7	581.7
1.3....	88 592.4	687.5	88 437.1	686.3	88 281.8	685.1	88 126.5	683.9	87 971.2	682.6
1.4....	95 406.6	797.3	95 239.3	795.9	95 072.1	794.5	94 904.9	793.1	94 737.6	791.7
1.5....	102 220.6	915.3	102 041.4	913.7	101 862.2	912.1	101 683.1	910.4	101 503.9	908.8
1.6....	109 034.5	1041.3	108 843.4	1039.4	108 652.3	1037.6	108 461.2	1035.8	108 270.0	1034.0
1.7....	115 848.3	1175.6	115 645.2	1173.5	115 442.1	1171.5	115 239.0	1169.4	115 036.0	1167.3
1.8....	122 661.9	1318.0	122 446.8	1315.6	122 231.8	1313.3	122 016.8	1311.0	121 801.8	1308.7
1.9....	129 475.3	1468.5	129 248.3	1465.9	129 021.3	1463.3	128 794.4	1460.7	128 567.4	1458.2
2.0....	136 288.5	1627.1	136 049.6	1624.2	135 810.7	1621.4	135 571.8	1618.5	135 332.9	1615.7
2.1....	143 101.6	1793.9	142 850.7	1790.7	142 599.9	1787.6	142 349.0	1784.4	142 098.1	1781.3
2.2....	149 914.3	1968.8	149 651.6	1965.3	149 388.8	1961.9	149 126.0	1958.4	148 863.2	1955.0
2.3....	156 727	2152	156 452	2148	156 178	2144	155 903	2141	155 628	2137
2.4....	163 539	2343	163 253	2339	162 966	2335	162 679	2331	162 393	2327
2.5....	170 352	2542	170 053	2538	169 754	2533	169 456	2529	169 157	2524
2.6....	177 164	2750	176 853	2745	176 543	2740	176 232	2735	175 921	2730
2.7....	183 975	2965	183 653	2960	183 330	2955	183 008	2950	182 685	2944
2.8....	190 787	3189	190 452	3183	190 118	3178	189 783	3172	189 449	3167
2.9....	197 598	3421	197 251	3415	196 905	3409	196 558	3403	196 212	3397
3.0....	204 408	3661	204 050	3654	203 692	3648	203 334	3642	202 975	3635
3.1....	211 219	3909	210 849	3902	210 479	3895	210 108	3888	209 738	3881
3.2....	218 029	4165	217 647	4158	217 265	4150	216 883	4143	216 500	4136
3.3....	224 839	4429	224 445	4422	224 051	4414	223 657	4406	223 263	4398
3.4....	231 649	4702	231 243	4694	230 837	4685	230 431	4677	230 025	4669
3.5....	238 458	4982	238 040	4974	237 622	4965	237 204	4956	236 786	4948
3.6....	245 267	5271	244 837	5262	244 407	5253	243 977	5244	243 547	5234
3.7....	252 076	5568	251 634	5558	251 192	5549	250 750	5539	250 308	5529
3.8....	258 884	5873	258 430	5863	257 976	5853	257 522	5842	257 069	5832
3.9....	265 692	6186	265 226	6175	264 760	6165	264 294	6154	263 829	6143
4.0....	272 499	6507	272 022	6496	271 544	6485	271 066	6473	270 589	6462
4.1....	279 306	6837	278 817	6825	278 327	6813	277 838	6801	277 348	6789
4.2....	286 113	7174	285 612	7162	285 110	7149	284 609	7137	284 107	7124
4.3....	292 919	7520	292 406	7507	291 893	7494	291 379	7480	290 866	7467
4.4....	299 725	7874	299 200	7860	298 675	7846	298 149	7832	297 624	7818
4.5....	306 531	8236	305 993	8221	305 456	8207	304 919	8192	304 381	8178

TABLE III—Continued.

Longitude	Latitude 53.0G		53.1G		53.2G		53.3G		53.4G	
	x	*y*	*x*	*y*	*x*	*y*	*x*	*y*	*x*	*y*
Grades	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*
0.02...	1351.1	0.2	1348.7	0.2	1346.3	0.2	1343.9	0.2	1341.5	0.2
0.04...	2702.1	0.6	2697.4	0.6	2692.6	0.6	2687.8	0.6	2683.0	0.6
0.06...	4053.2	1.5	4046.0	1.4	4038.9	1.4	4031.7	1.4	4024.5	1.4
0.08...	5404.3	2.6	5394.7	2.6	5385.2	2.6	5375.6	2.6	5366.0	2.6
0.10...	6755.3	4.0	6743.4	4.0	6731.4	4.0	6719.5	4.0	6707.6	4.0
0.12...	8106.4	5.8	8092.1	5.8	8077.7	5.8	8063.4	5.8	8049.1	5.8
0.14...	9457.5	7.9	9440.7	7.9	9424.0	7.9	9407.3	7.9	9390.6	7.8
0.16...	10 808.5	10.3	10 789.4	10.3	10 770.3	10.3	10 751.2	10.3	10 732.1	10.3
0.18...	12 159.6	13.1	12 138.1	13.0	12 116.6	13.0	12 095.1	13.0	12 073.6	13.1
0.2....	13 510.7	16.1	13 486.8	16.1	13 462.9	16.1	13 439.0	16.0	13 415.1	16.0
0.3....	20 266.0	36.3	20 230.1	36.2	20 194.3	36.2	20 158.5	36.1	20 122.6	36.0
0.4....	27 021.3	64.5	26 973.5	64.4	26 925.7	64.3	26 877.9	64.2	26 830.1	64.1
0.5....	33 776.5	100.8	33 716.8	100.6	33 657.0	100.5	33 597.3	100.3	33 537.6	100.1
0.6....	40 531.7	145.2	40 460.0	144.9	40 388.3	144.6	40 316.7	144.4	40 245.0	144.1
0.7....	47 286.8	197.6	47 203.2	197.2	47 119.6	196 9	47 036.0	196.5	46 952.4	196.2
0.8....	54 041.9	258.1	53 946.3	257.6	53 850.8	257.2	53 755.2	256.7	53 659.7	256.2
0.9....	60 796.9	326.6	60 689.4	326.0	60 581.9	325.5	60 474.4	324.9	60 366.9	324.3
1.0....	67 551.8	403.2	67 432.4	402.5	67 312.9	401.8	67 193.5	401.1	67 074.0	400.4
1.1....	74 306.6	487.9	74 175.2	487.0	74 043.8	486.2	73 912.4	485.3	73 781.0	484.4
1.2....	81 061.3	580.6	80 918.0	579.6	80 774.6	578.6	80 631.3	577.6	80 488.0	576.5
1.3....	87 815.9	681.4	87 660.6	680.2	87 505.3	679.0	87 350.0	677.8	87 194.7	676.6
1.4....	94 570.4	790.3	94 403.1	788.9	94 235.9	787.5	94 068.7	786.1	93 901.4	784.7
1.5....	101 324.7	907.2	101 145.5	905.6	100 966.3	904.0	100 787.2	902.4	100 608.0	900.8
1.6....	108 078.9	1032.2	107 887.8	1030.4	107 696.6	1028.6	107 505.5	1026.8	107 314.4	1024.9
1.7....	114 832.9	1165.3	114 629.8	1163.2	114 426.7	1161.2	114 223.7	1159.1	114 020.6	1157.0
1.8....	121 586.8	1306.4	121 371.8	1304.1	121 156.7	1301.8	120 941.8	1299.5	120 726.8	1297.2
1.9....	128 340.5	1455.6	128 113.5	1453.0	127 886.5	1450.4	127 659.6	1447.9	127 432.7	1445.3
2.0....	135 094.0	1612.8	134 855.1	1610.0	134 616.2	1607.1	134 377.3	1604.3	134 138.4	1601.4
2.1....	141 847.3	1778.1	141 596.4	1775.0	141 345.5	1771.9	141 094.8	1768.7	140 844.0	1765.6
2.2....	148 600.4	1951.5	148 337.6	1948.1	148 074.8	1944.6	147 812.1	1941.2	147 549.3	1937.7
2.3....	155 353	2133	155 079	2129	154 804	2125	154 529	2122	154 254	2118
2.4....	162 106	2322	161 819	2318	161 533	2314	161 246	2310	160 959	2306
2.5....	168 858	2520	168 560	2516	168 261	2511	167 963	2507	167 664	2502
2.6....	175 611	2726	175 300	2721	174 990	2716	174 679	2711	174 369	2706
2.7....	182 363	2939	182 040	2934	181 718	2929	181 395	2924	181 073	2918
2.8....	189 114	3161	188 780	3155	188 445	3150	188 111	3144	187 777	3139
2.9....	195 866	3391	195 519	3385	195 173	3379	194 827	3373	194 480	3367
3.0....	202 617	3629	202 259	3622	201 900	3616	201 542	3609	201 184	3603
3.1....	209 368	3875	208 997	3868	208 627	3861	208 257	3854	207 887	3847
3.2....	216 118	4129	215 736	4121	215 354	4114	214 972	4107	214 590	4099
3.3....	222 869	4391	222 474	4383	222 080	4375	221 686	4367	221 292	4360
3.4....	229 618	4661	229 212	4652	228 806	4644	228 400	4636	227 994	4628
3.5....	236 368	4939	235 950	4930	235 532	4921	235 114	4913	234 696	4904
3.6....	243 117	5225	242 687	5216	242 258	5207	241 828	5197	241 398	5188
3.7....	249 866	5519	249 424	5510	248 983	5500	248 541	5490	248 099	5480
3.8....	256 615	5822	256 161	5811	255 707	5801	255 254	5791	254 800	5780
3.9....	263 363	6132	262 897	6121	262 432	6110	261 966	6100	261 500	6089
4.0....	270 111	6450	269 633	6439	269 156	6428	268 678	6416	268 200	6405
4.1....	276 858	6777	276 369	6765	275 879	6753	275 390	6741	274 900	6729
4.2....	283 605	7111	283 104	7099	282 602	7086	282 101	7074	281 599	7061
4.3....	290 352	7454	289 839	7441	289 325	7428	288 812	7414	288 298	7401
4.4....	297 098	7805	296 573	7791	296 048	7777	295 522	7763	294 997	7750
4.5....	303 844	8163	303 307	8149	302 770	8135	302 232	8120	301 695	8106

TABLE III—Continued.

Longitude	Latitude 53.5°		53.6°		53.7°		53.8°		53.9°	
	x	y	x	y	x	y	x	y	x	y
Grades	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*
0.02...	1339.1	0.2	1336.7	0.2	1334.3	0.2	1332.0	0.2	1329.6	0.2
0.04...	2678.2	0.6	2673.5	0.6	2668.7	0.6	2663.9	0.6	2659.1	0.6
0.06...	4017.4	1.4	4010.2	1.4	4003.1	1.4	3995.9	1.4	3988.8	1.4
0.08...	5356.5	2.6	5346.9	2.6	5337.4	2.5	5327.8	2.5	5318.2	2.5
0.10...	6695.6	4.0	6683.7	4.0	6671.7	4.0	6659.8	4.0	6647.9	4.0
0.12...	8034.7	5.8	8020.4	5.7	8006.1	5.7	7991.7	5.7	7977.4	5.7
0.14...	9373.9	7.8	9357.1	7.8	9340.4	7.8	9323.7	7.8	9307.0	7.8
0.16...	10 713.0	10.2	10 693.9	10.2	10 674.8	10.2	10 655.7	10.2	10 636.6	10.2
0.18...	12 052.1	12.9	12 030.6	12.9	12 009.1	12.9	11 987.6	12.9	11 966.1	12.9
0.2....	13 391.2	16.0	13 367.3	16.0	13 343.4	15.9	13 319.5	15.9	13 295.6	15.9
0.3....	20 086.8	36.0	20 050.9	35.9	20 015.1	35.8	19 979.3	35.8	19 943.4	35.7
0.4....	26 782.4	63.9	26 734.6	63.8	26 686.8	63.7	26 639.0	63.6	26 591.2	63.5
0.5....	33 477.9	99.9	33 418.2	99.7	33 358.4	99.6	33 298.7	99.4	33 239.0	99.2
0.6....	40 173.3	143.9	40 101.6	143.6	40 029.9	143.4	39 958.3	143.1	39 886.6	142.8
0.7....	46 868.8	195.8	46 785.2	195.5	46 701.6	195.1	46 618.0	194.8	46 534.4	194.4
0.8....	53 564.1	255.8	53 468.5	255.3	53 373.0	254.9	53 277.4	254.4	53 181.9	254.0
0.9....	60 259.4	323.7	60 151.9	323.1	60 044.4	322.6	59 936.9	322.0	59 829.4	321.4
1.0....	66 954.6	399.7	66 835.2	398.9	66 715.7	398.2	66 596.3	397.5	66 476.8	396.8
1.1....	73 649.6	483.6	73 518.2	482.7	73 386.8	481.9	73 255.4	481.0	73 124.0	480.1
1.2....	80 344.6	575.5	80 201.3	574.5	80 057.9	573.4	79 914.6	572.4	79 771.3	571.4
1.3....	87 039.5	675.4	86 884.2	674.2	86 728.9	673.0	86 573.6	671.8	86 418.3	670.6
1.4....	93 734.2	783.3	93 567 0	781.9	93 399.8	780.5	93 232.6	779.1	93 065.4	777.7
1.5....	100 428.8	899.2	100 249.7	897.6	100 070.5	896.0	99 891.4	894.4	199 712.2	892.8
1.6....	107 123.3	1023.1	106 932.2	1021.3	106 741.1	1019.4	106 550.0	1017.6	106 358.9	1015.8
1.7....	113 817.6	1155.0	113 611.5	1152.9	113 411.5	1150.9	113 208.5	1148.8	113 005.4	1146.7
1.8....	120 511.8	1294.9	120 296.8	1292.6	120 081.8	1290.2	119 866.8	1287.9	119 651.8	1285.6
1.9....	127 205.7	1442.7	126 978.8	1440.2	126 751.9	1437.6	126 525.0	1435.0	126 298.1	1432.4
2.0....	133 899.5	1598.6	133 660.7	1595.7	133 421.9	1592.9	133 183.0	1590.0	132 944.2	1587.2
2.1....	140 593.2	1762.4	140 342.4	1759.3	140 091.5	1756.1	139 840.7	1753.0	139 589.9	1749.8
2.2....	147 286.5	1934.3	147 023.8	1930.8	146 761.0	1927.4	146 498.3	1923.9	146 235.5	1920.4
2.3....	153 980	2114	153 705	2110	153 430	2106	153 155	2103	152 880	2099
2.4....	160 673	2302	160 386	2298	160 100	2294	159 813	2290	159 526	2285
2.5....	167 365	2498	167 067	2493	166 768	2489	166 470	2484	166 171	2480
2.6....	174 058	2702	173 747	2697	173 437	2692	173 126	2687	172 816	2682
2.7....	180 750	2913	180 427	2908	180 105	2903	179 783	2898	179 461	2892
2.8....	187 442	3133	187 108	3128	186 774	3122	186 439	3116	186 105	3111
2.9....	194 134	3361	193 787	3355	193 441	3349	193 095	3343	192 748	3337
3.0....	200 825	3597	200 468	3590	200 109	3584	199 751	3577	199 393	3571
3.1....	207 517	3840	207 147	3833	206 776	3827	206 406	3820	206 036	3813
3.2....	214 208	4092	213 826	4085	213 444	4077	213 062	4070	212 680	4063
3.3....	220 898	4352	220 503	4344	220 109	4336	219 715	4328	219 321	4321
3.4....	227 588	4620	227 182	4611	226 776	4603	226 370	4595	225 964	4586
3.5....	234 278	4895	233 860	4886	233 442	4878	233 024	4869	232 606	4860
3.6....	240 968	5179	240 538	5170	240 109	5160	239 679	5151	239 249	5142
3.7....	247 657	5470	247 215	5461	246 774	5451	246 332	5441	245 890	5432
3.8....	254 346	5770	253 892	5760	253 438	5750	252 985	5739	252 531	5729
3.9....	261 035	6078	260 569	6067	260 103	6056	259 637	6045	259 171	6034
4.0....	267 723	6393	267 245	6382	266 768	6370	266 290	6359	265 812	6348
4.1....	274 411	6717	271 922	6705	273 432	6693	272 943	6681	272 453	6669
4.2....	281 098	7048	280 597	7036	280 095	7023	279 594	7011	279 092	6998
4.3....	287 785	7388	287 272	7375	286 758	7362	286 245	7349	285 731	7335
4.4....	294 472	7736	293 946	7722	293 421	7708	292 895	7694	292 370	7680
4.5....	301 158	8091	300 620	8077	300 083	8062	299 546	8048	299 009	8033

TABLE III—Continued.

Longitude	Latitude 54.0ᴳ		54.1ᴳ		54.2ᴳ		54.3ᴳ		54.4ᴳ	
	x	y	x	y	x	y	x	y	x	y
Grades	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*
0.02...	1327.2	0.2	1324.8	0.2	1322.4	0.2	1320.0	0.2	1317.6	0.2
0.04...	2654.4	0.6	2649.6	0.6	2644.8	0.6	2640.0	0.6	2635.2	0.6
0.06...	3981.6	1.4	3974.4	1.4	3967.2	1.4	3960.0	1.4	3952.9	1.4
0.08...	5308.7	2.5	5299.2	2.5	5289.6	2.5	5280.0	2.5	5270.5	2.5
0.10...	6635.9	4.0	6624.0	4.0	6612.0	3.9	6600.1	3.9	6588.1	3.9
0.12...	7963.1	5.7	7948.7	5.7	7934.4	5.7	7920.1	5.7	7905.7	5.7
0.14...	9290.3	7.8	9273.5	7.7	9256.8	7.7	9240.1	7.7	9223.4	7.7
0.16...	10 617.5	10.1	10 598.3	10.1	10 579.2	10.1	10 560.1	10.1	10 541.0	10.1
0.18...	11 944.6	12.8	11 923.1	12.8	11 901.6	12.8	11 880.1	12.8	11 858.6	12.7
0.2....	13 271.7	15.8	13 247.9	15.8	13 224.0	15.8	13 200.1	15.8	13 176.2	15.7
0.3....	19 907.6	35.6	19 871.8	35.6	19 836.0	35.5	19 800.2	35.5	19 764.3	35.4
0.4....	26 543.5	63.4	26 495.7	63.3	26 447 9	63.1	26 400.2	63.0	26 352.4	62.9
0.5....	33 179.3	99.0	33 119.6	98.8	33 059.8	98.7	33 000.1	98.5	32 940.4	98.3
0.6....	39 814.9	142.6	39 743.4	142.3	39 671.7	142.1	39 600.0	141.8	39 528.4	141.6
0.7....	46 450.8	194.1	46 367.1	193.7	46 283.5	193.4	46 199.9	193.0	46 116.3	192.7
0.8....	53 086.3	253.5	52 990.8	253.0	52 895.3	252.6	52 799.7	252.1	52 704.2	251.7
0.9....	59 721.9	320.8	59 614.4	320.3	59 506.9	319.7	59 399.4	319.1	59 292.0	318.5
1.0....	66 357.4	396.1	66 237.9	395.4	66 118.5	394.7	65 999.1	394.0	65 879.6	393.2
1.1....	72 992.7	479.3	72 861.4	478.4	72 730.0	477.5	72 598.6	476.7	72 467.2	475.8
1.2....	79 628.0	570.4	79 484.7	569.3	79 341.4	568.3	79 198.1	567.3	79 054.8	566.3
1.3....	86 263.1	669.4	86 107.9	668.2	85 952.6	667.0	85 797.4	665.8	85 642.1	664.6
1.4....	92 898.2	776.3	92 731.0	774.9	92 563.8	773.5	92 396.6	772.1	92 229.4	770.7
1.5....	99 533.0	891.2	99 353.9	889.6	99 174.8	888.0	98 995.7	886.4	98 816.5	884.8
1.6....	106 167.8	1014.0	105 976.8	1012.2	105 785.7	1010.3	105 594.6	1008.5	105 403.5	1006.7
1.7....	112 802.4	1144.7	112 599.4	1142.6	112 396.4	1140.6	112 193.4	1138.5	111 990.3	1136.4
1.8....	119 436.9	1283.3	119 222.0	1281.0	119 007.0	1278.7	118 792.0	1276.4	118 577.1	1274.1
1.9....	126 071.2	1429.8	125 844.3	1427.3	125 617.4	1424.7	125 390.5	1422.1	125 163.6	1419.6
2.0....	132 705.3	1584.3	132 466.4	1581.5	132 227.6	1578.6	131 988.7	1575.8	131 749.9	1572.9
2.1....	139 339.1	1746.7	139 088.4	1743.6	138 837.6	1740.4	138 586.8	1737.3	138 336.1	1734.1
2.2....	145 972.8	1917.0	145 710.1	1913.6	145 447.4	1910.1	145 184.7	1906.6	144 922.0	1903.2
2.3....	152 606	2095	152 332	2091	152 057	2088	151 782	2084	151 508	2080
2.4....	159 240	2281	158 953	2277	158 666	2273	158 380	2269	158 093	2265
2.5....	165 872	2475	165 574	2471	165 275	2466	164 977	2462	164 679	2458
2.6....	172 505	2677	172 195	2673	171 885	2668	171 574	2663	171 264	2658
2.7....	179 138	2887	178 816	2882	178 493	2877	178 171	2872	177 848	2866
2.8....	185 770	3105	185 436	3100	185 102	3094	184 767	3088	184 433	3083
2.9....	192 402	3331	192 056	3325	191 710	3319	191 364	3313	191 017	3307
3.0....	199 035	3564	198 676	3558	198 317	3552	197 960	3545	197 601	3539
3.1....	205 666	3806	205 296	3799	204 925	3792	204 555	3786	204 185	3779
3.2....	212 298	4056	211 915	4048	211 533	4041	211 151	4034	210 769	4026
3.3....	218 927	4313	218 534	4305	218 140	4297	217 746	4290	217 352	4282
3.4....	225 558	4578	225 152	4570	224 747	4562	224 340	4254	223 935	4545
3.5....	232 188	4852	231 771	4843	231 353	4834	230 935	4825	230 517	4817
3.6....	238 819	5133	238 389	5123	237 959	5114	237 529	5105	237 099	5096
3.7....	245 448	5422	245 006	5412	244 565	5402	244 123	5392	243 681	5383
3.8....	252 078	5719	251 624	5708	251 170	5698	250 716	5688	250 263	5678
3.9....	258 706	6024	258 241	6013	257 775	6002	257 310	5991	256 844	5980
4.0....	265 335	6336	264 857	6325	264 380	6314	263 902	6302	263 425	6291
4.1....	271 964	6657	271 474	6645	270 984	6633	270 495	6621	270 005	6609
4.2....	278 591	6986	278 089	6973	277 588	6960	277 086	6948	276 585	6935
4.3....	285 218	7322	284 705	7309	284 192	7296	283 678	7283	283 165	7270
4.4....	291 845	7667	291 320	7653	290 795	7639	290 269	7625	289 744	7612
4.5....	298 472	8019	297 934	8005	297 397	7990	296 860	7976	296 323	7961

TABLE III—Continued.

Longitude	Latitude 54.5ᴳ		54.6ᴳ		54.7ᴳ		54.8ᴳ		54.9ᴳ	
	x	*y*	*x*	*y*	*x*	*y*	*x*	*y*	*x*	*y*
Grades	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*
0.02....	1315.2	0.2	1312.8	0.2	1310.5	0.2	1308.1	0.2	1305.7	0.2
0.04....	2630.5	0.6	2625.7	0.6	2620.9	0.6	2616.1	0.6	2611.4	0.6
0.06....	3945.7	1.4	3938.5	1.4	3931.4	1.4	3924.2	1.4	3917.1	1.4
0.08....	5260.9	2.5	5251.4	2.5	5241.8	2.5	5232.3	2.5	5222.8	2.5
0.10....	6576.2	3.9	6564.2	3.9	6552.3	3.9	6540.3	3.9	6528.4	3.9
0.12....	7891.4	5.6	7877.1	5.6	7862.7	5.6	7848.4	5.6	7834.1	5.6
0.14....	9206.6	7.7	9189.9	7.7	9173.2	7.7	9156.5	7.6	9139.8	7.6
0.16....	10 521.9	10.0	10 502.8	10.0	10 483.7	10.0	10 464.6	10.0	10 445.5	10.0
0.18....	11 837.1	12.7	11 815.6	12.7	11 794.1	12.7	11 772.6	12.6	11 751.1	12.6
0.2....	13 152.3	15.7	13 128.4	15.7	13 104.6	15.6	13 080.7	15.6	13 056.8	15.6
0.3....	19 728.5	35.3	19 692.7	35.3	19 656.8	35.2	19 621.0	35.1	19 585.2	35.1
0.4....	26 304.6	62.8	26 256.8	62.7	26 209.1	62.6	26 161.3	62.5	26 113.5	62.3
0.5....	32 880.7	98.1	32 821.0	98.0	32 761.3	97.8	32 701.5	97.6	32 641.8	97.4
0.6....	39 456.7	141.3	39 385.1	141.1	39 313.4	140.8	39 241.8	140.5	39 170.1	140.3
0.7....	46 032.7	192.3	45 949.1	192.0	45 865.5	191.6	45 781.9	191.3	45 698.3	190.9
0.8....	52 608.6	251.2	52 513.1	250.8	52 417.5	250.3	52 322.0	249.8	52 226.4	249.4
0.9....	59 184.5	318.0	59 077.0	317.4	58 969.5	316.8	58 862.0	316.2	58 754.5	315.6
1.0....	65 760.2	392.5	65 640.8	391.8	65 521.4	391.1	65 401.9	390.4	65 282.5	389.7
1.1....	72 335.9	475.0	72 204.5	474.1	72 073.1	473.2	71 941.8	472.4	71 810.4	471.5
1.2....	78 911.4	565.2	78 768.1	564.2	78 624.8	563.2	78 481.5	562.2	78 338.2	561.1
1.3....	85 486.9	663.4	85 331.6	662.2	85 176.4	661.0	85 021.1	659.8	84 865.8	658.5
1.4....	92 062.2	769.3	91 895.0	768.0	91 727.8	766.6	91 560.6	765.2	91 393.4	763.8
1.5....	98 637.4	883.2	98 458.2	881.6	98 279.1	880.0	98 100.0	878.4	97 920.8	876.8
1.6....	105 212.5	1004.8	105 021.4	1003.0	104 830.3	1001.2	104 639.2	999.4	104 448.1	997.6
1.7....	111 787.3	1134.4	111 584.3	1132.3	111 381.3	1130.3	111 178.3	1128.2	110 975.2	1126.1
1.8....	118 362.1	1271.8	118 147.1	1269.4	117 932.2	1267.1	117 717.2	1264.8	117 502.2	1262.5
1.9....	124 936.7	1417.0	124 709.8	1414.4	124 482.9	1411.8	124 256.0	1409.3	124 029.1	1406.7
2.0....	131 511.1	1570.1	131 272.2	1567.2	131 033.4	1564.4	130 794.5	1561.5	130 555.7	1558.7
2.1....	138 085.3	1731.0	137 834.5	1727.8	137 583.7	1724.7	137 332.9	1721.6	137 082.1	1718.4
2.2....	144 659.3	1899.8	144 396.6	1896.3	144 133.8	1892.8	143 871.1	1889.4	143 608.4	1886.0
2.3....	151 233	2076	150 958	2073	150 684	2069	150 409	2065	150 134	2061
2.4....	157 807	2261	157 520	2257	157 233	2253	156 947	2248	156 660	2244
2.5....	164 380	2453	164 082	2449	163 783	2444	163 484	2440	163 186	2435
2.6....	170 953	2653	170 643	2648	170 332	2644	170 022	2639	169 711	2634
2.7....	177 526	2861	177 204	2856	176 881	2851	176 559	2846	176 236	2841
2.8....	184 099	3077	183 764	3072	183 430	3066	183 096	3060	182 761	3055
2.9....	190 671	3301	190 325	3295	189 978	3289	189 632	3283	189 286	3277
3.0....	197 243	3532	196 885	3526	196 527	3520	196 168	3513	195 810	3507
3.1....	203 815	3772	203 445	3765	203 075	3758	202 704	3751	202 334	3744
3.2....	210 386	4019	210 004	4012	209 622	4004	209 240	3997	208 858	3990
3.3....	216 958	4274	216 564	4266	216 170	4259	215 776	4251	215 382	4243
3.4....	223 529	4537	223 123	4529	222 717	4521	222 311	4512	221 905	4504
3.5....	230 099	4808	229 681	4799	229 263	4790	228 846	4782	228 428	4773
3.6....	236 670	5086	236 240	5077	235 810	5069	235 380	5059	234 950	5050
3.7....	243 240	5373	242 798	5363	242 356	5353	241 914	5344	241 472	5334
3.8....	249 809	5667	249 355	5657	248 902	5647	248 448	5636	247 994	5626
3.9....	256 378	5969	255 913	5958	255 447	5948	254 981	5937	254 516	5926
4.0....	262 947	6279	262 470	6267	261 992	6256	261 514	6245	261 037	6234
4.1....	269 516	6597	269 026	6585	268 537	6573	268 047	6561	267 558	6549
4.2....	276 084	6923	275 582	6910	275 081	6898	274 580	6885	274 078	6873
4.3....	282 652	7256	282 138	7243	281 625	7230	281 112	7217	280 598	7204
4.4....	289 219	7598	288 694	7584	288 168	7570	287 643	7556	287 118	7543
4.5....	295 786	7947	295 248	7932	294 711	7918	294 174	7904	293 637	7889

TABLE III—Continued.

Longitude	Latitude 55.0ᴳ		55.1ᴳ		55.2ᴳ		55.3ᴳ		55.4ᴳ	
	x	y	x	y	x	y	x	y	x	y
Grades	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*
0.02...	1303.3	0.2	1300.9	0.2	1298.5	0.2	1296.1	0.2	1293.8	0.2
0.04...	2606.6	0.6	2601.8	0.6	2597.0	0.6	2592.3	0.6	2587.5	0.6
0.06...	3909.9	1.4	3902.7	1.4	3895.6	1.4	3888.4	1.4	3881.2	1.4
0.08...	5213.2	2.5	5203.6	2.5	5194.1	2.5	5184.5	2.5	5175.0	2.5
0.10...	6516.5	3.9	6504.5	3.9	6492.6	3.9	6480.6	3.9	6468.7	3.9
0.12...	7819.8	5.6	7805.5	5.6	7791.1	5.6	7776.8	5.6	7762.4	5.6
0.14...	9123.1	7.6	9106.4	7.6	9089.6	7.6	9072.9	7.6	9056.2	7.6
0.16...	10 426.4	10.0	10 407.3	9.9	10 388.1	9.9	10 369.0	9.9	10 349.9	9.9
0.18...	11 729.6	12.6	11 708.1	12.6	11 686.6	12.6	11 665.1	12.5	11 643.6	12.5
0.2....	13 032.9	15.6	13 009.0	15.5	12 985.1	15.5	12 961.2	15.5	12 937.4	15.4
0.3....	19 549.3	35.0	19 513.5	34.9	19 477.7	34.9	19 441.8	34.8	19 406.0	34.8
0.4....	26 065.7	62.2	26 018.0	62.1	25 970.2	62.0	25 922.4	61.9	25 874.6	61.8
0.5....	32 582.1	97.2	32 522.4	97.1	32 462.7	96.9	32 402.9	96.7	32 343.2	96.5
0.6....	39 098.4	140.0	39 026.8	139.8	38 955.1	139.5	38 883.4	139.3	38 811.8	139.0
0.7....	45 614.7	190.6	45 531.1	190.2	45 447.5	189.9	45 363.9	189.5	45 280.3	189.2
0.8....	52 130.9	248.9	52 035.3	248.5	51 939.8	248.0	51 844.2	247.6	51 748.7	247.1
0.9....	58 647.0	315.1	58 539.5	314.5	58 432.0	313.9	58 324.5	313.3	58 217.0	312.8
1.0....	65 163.1	389.0	65 043.6	388.3	64 924.2	387.5	64 804.7	386.8	64 685.3	386.1
1.1....	71 679.0	470.6	71 547.6	469.8	71 416.2	468.9	71 284.8	468.1	71 153.5	467.2
1.2....	78 194.9	560.1	78 051.5	559.1	77 908.2	558.1	77 764.9	557.0	77 621.6	556.0
1.3....	84 710.6	657.3	84 555.3	656.1	84 400.0	654.9	84 244.7	653.7	84 089.5	652.5
1.4....	91 226.2	762.4	91 059.0	761.0	90 891.8	759.6	90 724.5	758.2	90 557.3	756.8
1.5....	97 741.7	875.2	97 562.5	873.6	97 383.4	871.9	97 204.2	870.3	97 025.0	868.7
1.6....	104 257.0	995.7	104 065.9	993.9	103 874.8	992.1	103 683.7	990.3	103 492.6	988.4
1.7....	110 772.2	1124.1	110 569.2	1122.0	110 366.1	1120.0	110 163.1	1117.9	109 960.0	1115.8
1.8....	117 287.3	1260.2	117 072.3	1257.9	116 857.3	1255.6	116 642.3	1253.3	116 427.3	1251.0
1.9....	123 802.1	1404.1	123 575.2	1401.5	123 348.3	1399.0	123 121.3	1396.4	122 894.4	1393.8
2.0....	130 316.8	1555.8	130 078.0	1552.9	129 839.1	1550.1	129 600.2	1547.2	129 361.4	1544.4
2.1....	136 831.3	1715.3	136 580.5	1712.1	136 329.7	1709.0	136 078.9	1705.8	135 828.1	1702.7
2.2....	143 345.6	1882.5	143 082.9	1879.1	142 820.1	1875.6	142 557.4	1872.1	142 294.6	1868.7
2.3....	149 860	2058	149 585	2054	149 310	2050	149 036	2046	148 761	2042
2.4....	156 374	2240	156 087	2236	155 800	2232	155 514	2228	155 227	2224
2.5....	162 887	2431	162 589	2426	162 290	2422	161 992	2418	161 693	2413
2.6....	169 401	2629	169 090	2624	168 780	2620	168 469	2615	168 159	2610
2.7....	175 914	2835	175 592	2830	175 269	2825	174 947	2820	174 624	2815
2.8....	182 427	3049	182 093	3044	181 758	3038	181 424	3032	181 089	3027
2.9....	188 940	3271	188 593	3265	188 247	3259	187 901	3253	187 554	3247
3.0....	195 452	3500	195 094	3494	194 735	3488	194 377	3481	194 019	3475
3.1....	201 964	3738	201 594	3731	201 224	3724	200 854	3717	200 483	3710
3.2....	208 476	3983	208 094	3975	207 712	3968	207 330	3961	206 947	3953
3.3....	214 988	4235	214 593	4228	214 199	4220	213 805	4212	213 411	4204
3.4....	221 499	4496	221 093	4488	220 687	4479	220 281	4471	219 875	4463
3.5....	228 010	4764	227 592	4755	227 174	4747	226 756	4738	226 338	4729
3.6....	234 520	5040	234 091	5031	233 661	5022	233 231	5013	232 801	5003
3.7....	241 031	5324	240 589	5314	240 147	5305	239 705	5295	239 263	5285
3.8....	247 540	5616	247 087	5605	246 633	5595	246 179	5585	245 726	5575
3.9....	254 050	5915	253 584	5904	253 119	5893	252 653	5883	252 187	5872
4.0....	260 559	6222	260 082	6211	259 604	6199	259 127	6187	258 649	6176
4.1....	267 068	6537	266 579	6525	266 089	6513	265 600	6501	265 110	6489
4.2....	273 577	6860	273 075	6847	272 574	6835	272 072	6822	271 571	6810
4.3....	280 085	7190	279 571	7177	279 058	7164	278 545	7151	278 031	7138
4.4....	286 593	7529	286 067	7515	285 542	7501	285 017	7487	284 491	7474
4.5....	293 100	7875	292 562	7860	292 025	7846	291 488	7831	290 950	7817

TABLE III—Continued.

Longitude	Latitude 55.5°		55.6°		55.7°		55.8°		55.9°	
	x	*y*	*x*	*y*	*x*	*y*	*x*	*y*	*x*	*y*
Grades	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*
0.02	1291.4	0.2	1289.0	0.2	1286.6	0.2	1284.2	0.2	1281.8	0.2
0.04	2582.7	0.6	2577.9	0.6	2573.1	0.6	2568.4	0.6	2563.6	0.6
0.06	3874.1	1.4	3866.9	1.4	3859.7	1.4	3852.5	1.4	3845.4	1.4
0.08	5165.4	2.5	5155.8	2.5	5146.3	2.5	5136.7	2.5	5127.2	2.4
0.10	6456.8	3.9	6444.8	3.8	6432.8	3.8	6420.9	3.8	6408.9	3.8
0.12	7748.1	5.6	7733.8	5.5	7719.4	5.5	7705.1	5.5	7690.7	5.5
0.14	9039.4	7.6	9022.7	7.5	9006.0	7.5	8989.3	7.5	8972.5	7.5
0.16	10 330.8	9.9	10 311.7	9.8	10 292.5	9.8	10 273.4	9.8	10 254.3	9.8
0.18	11 622.1	12.5	11 600.6	12.5	11 579.1	12.4	11 557.6	12.4	11 536.1	12.4
0.2	12 913.5	15.4	12 889.6	15.4	12 865.7	15.4	12 841.8	15.3	12 817.9	15.3
0.3	19 370.2	34.7	19 334.3	34.6	19 298.5	34.6	19 262.7	34.5	19 226.8	34.4
0.4	25 826.8	61.7	25 779.1	61.6	25 731.3	61.4	25 683.5	61.3	25 635.7	61.2
0.5	32 283.5	96.4	32 223.8	96.2	32 164.0	96.0	32 104.3	95.8	32 044.6	95.6
0.6	38 740.1	138.7	38 668.4	138.5	38 596.8	138.2	38 525.1	138.0	38 453.4	137.7
0.7	45 196.6	188.8	45 113.0	188.5	45 029.4	188.1	44 945.8	187.8	44 862.1	187.5
0.8	51 653.1	246.7	51 557.6	246.2	51 462.0	245.7	51 366.4	245.3	51 270.8	244.8
0.9	58 109.5	312.2	58 002.0	311.6	57 894.5	311.0	57 787.0	310.4	57 679.5	309.9
1.0	64 565.8	385.4	64 446.4	384.7	64 326.9	384.0	64 207.5	383.3	64 088.0	382.5
1.1	71 022.1	466.3	70 890.7	465.5	70 759.3	464.6	70 627.9	463.7	70 496.4	462.9
1.2	77 478.2	555.0	77 334.9	553.9	77 191.5	552.9	77 048.2	551.9	76 904.8	550.9
1.3	83 934.2	651.3	83 778.9	650.1	83 623.6	648.9	83 468.3	647.7	83 313.0	646.5
1.4	90 390.1	755.4	90 222.9	754.0	90 055.7	752.6	89 888.4	751.2	89 721.1	749.8
1.5	96 845.9	867.1	96 666.7	865.5	96 487.5	863.9	96 308.3	862.3	96 129.1	860.7
1.6	103 301.5	986.6	103 110.4	984.8	102 919.3	983.0	102 728.1	981.1	102 537.0	979.3
1.7	109 757.0	1113.8	109 553.9	1111.7	109 350.9	1109.7	109 147.8	1107.6	108 944.7	1105.5
1.8	116 212.3	1248.7	115 997.4	1246.4	115 782.3	1244.0	115 567.3	1241.7	115 352.3	1239.4
1.9	122 667.5	1391.3	122 440.6	1388.7	122 213.6	1386.1	121 986.6	1383.5	121 759.6	1381.0
2.0	129 122.5	1541.5	128 883.6	1538.7	128 644.7	1535.8	128 405.8	1533.0	128 166.9	1530.1
2.1	135 577.3	1699.5	135 326.5	1696.4	135 075.6	1693.3	134 824.8	1690.1	134 573.9	1687.0
2.2	142.031.9	1865.2	141 769.2	1861.8	141 506.4	1858.3	141 243.5	1854.9	140 980.7	1851.4
2.3	148 486	2039	148 212	2035	147 937	2031	147 662	2027	147 387	2024
2.4	154 940	2220	154 654	2216	154 367	2212	154 080	2207	153 794	2203
2.5	161 394	2409	161 096	2404	160 797	2400	160 499	2395	160 200	2391
2.6	167 848	2605	167 538	2600	167 227	2595	166 917	2591	166 606	2586
2.7	174 302	2809	173 979	2804	173 657	2799	173 334	2794	173 012	2789
2.8	180 755	3021	180 421	3016	180 086	3010	179 752	3005	179 417	2999
2.9	187 208	3241	186 862	3235	186 515	3229	186 169	3223	185 823	3217
3.0	193 661	3468	193 302	3462	192 944	3455	192 586	3449	192 227	3443
3.1	200 113	3703	199 743	3696	199 373	3690	199 002	3683	198 632	3676
3.2	206 565	3946	206 183	3939	205 801	3931	205 419	3924	205 037	3917
3.3	213 017	4197	212 623	4189	212 229	4181	211 835	4173	211 441	4165
3.4	219 469	4455	219 063	4446	218 657	4438	218 251	4430	217 845	4422
3.5	225 920	4721	225 502	4712	225 084	4703	224 666	4694	224 248	4686
3.6	232 371	4994	231 941	4985	231 511	4976	231 081	4966	230 651	4957
3.7	238 822	5275	238 380	5266	237 938	5256	237 496	5246	237 054	5236
3.8	245 272	5564	244 818	5554	244 364	5544	243 910	5533	243 457	5523
3.9	251 722	5861	251 256	5850	250 790	5839	250 325	5828	249 859	5818
4.0	258 171	6164	257 694	6152	257 216	6142	256 738	6131	256 261	6120
4.1	264 621	6477	264 131	6465	263 641	6453	263 152	6441	262 662	6429
4.2	271 069	6797	270 568	6785	270 066	6772	269 565	6759	269 063	6747
4.3	277 518	7125	277 004	7111	276 491	7098	275 977	7085	275 464	7072
4.4	283 966	7460	283 441	7446	282 915	7432	282 390	7418	281 864	7404
4.5	290 413	7803	289 876	7788	289 339	7774	288 802	7759	288 264	7745

TABLE III—Continued.

Longitude	Latitude 56.0G		56.1G		56.2G		56.3G		56.4G	
	x	y	x	y	x	y	x	y	x	y
Grades	Meters	Meters	Meters	Meters	Meters	Meters	Meters	Meters	Meters	Meters
0.02....	1279.4	0.2	1277.0	0.2	1274.6	0.2	1272.2	0.2	1269.8	0.2
0.04....	2558.8	0.6	2554.0	0.6	2549.2	0.6	2544.5	0.6	2539.7	0.6
0.06....	3838.2	1.4	3831.0	1.4	3823.9	1.4	3816.7	1.4	3809.5	1.4
0.08....	5117.6	2.4	5108.0	2.4	5098.5	2.4	5088.9	2.4	5079.4	2.4
0.10....	6397.0	3.8	6385.1	3.8	6373.1	3.8	6361.2	3.8	6349.2	3.8
0.12....	7676.4	5.5	7662.1	5.5	7647.7	5.5	7633.4	5.5	7619.1	5.5
0.14....	8955.8	7.5	8939.1	7.5	8922.3	7.5	8905.6	7.4	8888.9	7.4
0.16....	10 235.2	9.8	10 216.1	9.8	10 197.0	9.7	10 177.9	9.7	10 158.7	9.7
0.18....	11 514.6	12.4	11 493.1	12.3	11 471.6	12.3	11 450.1	12.3	11 428.6	12.3
0.2.....	12 794.0	15.3	12 770.1	15.2	12 746.2	15.2	12 722.3	15.2	12 698.4	15.2
0.3.....	19 191.0	34.4	19 155.1	34.3	19 119.3	34.2	19 083.4	34.2	19 047.6	34.1
0.4.....	25 587.9	61.1	25 540.1	61.0	25 492.3	60.9	25 444.6	60.8	25 396.8	60.6
0.5.....	31 984.8	95.5	31 925.1	95.3	31 865.4	95.1	31 805.6	94.9	31 745.9	94.7
0.6.....	38 381.7	137.5	38 310.0	137.2	38 238.3	136.9	38 166.6	136.7	38 095.0	136.4
0.7.....	44 778.5	187.1	44 694.9	186.8	44 611.3	186.4	44 527.6	186.1	44 444.0	185.7
0.8.....	51 175.3	244.4	51 079.7	243.9	50 984.1	243.5	50 888.5	243.0	50 793.0	242.5
0.9.....	57 571.9	309.3	57 464.4	308.7	57 356.9	308.1	57 249.4	307.6	57 141.9	307.0
1.0.....	63 968.5	381.8	63 849.1	381.1	63 729.6	380.4	63 610.1	379.7	63 490.7	379.0
1.1.....	70 365.0	462.0	70 233.6	461.2	70 102.2	460.3	69 970.8	459.4	69 839.4	458.6
1.2.....	76 761.4	549.8	76 618.1	548.8	76 474.7	547.8	76 331.4	546.8	76 188.0	545.7
1.3.....	83 157.7	645.3	83 002.4	644.1	82 847.1	642.9	82 691.8	641.7	82 536.5	640.5
1.4.....	89 553.9	748.4	89 386.6	747.0	89 219.4	745.6	89 052.1	744.2	88 884.9	742.8
1.5.....	95 949.9	859.1	95 770.7	857.5	95 591.5	855.9	95 412.3	854.3	95 233.1	852.7
1.6.....	102 345.9	977.5	102 154.7	975.7	101 963.6	973.8	101 772.4	972.0	101 581.3	970.2
1.7.....	108 741.6	1103.5	108 538.5	1101.4	108 335.4	1099.4	108 132.3	1097.3	107 929.2	1095.2
1.8.....	115 137.2	1237.1	114 922.2	1234.8	114 707.2	1232.5	114 492.1	1230.2	114 277.1	1227.9
1.9.....	121 532.7	1378.4	121 305.7	1375.8	121 078.7	1373.2	120 851.7	1370.7	120 624.8	1368.1
2.0.....	127 927.9	1527.3	127 689.0	1524.4	127 450.1	1521.6	127 211.2	1518.7	126 972.3	1515.9
2.1.....	134 323.0	1683.8	134 072.2	1680.7	133 821.3	1677.5	133 570.5	1674.4	133 319.6	1671.2
2.2.....	140 717.9	1848.0	140 455.1	1844.5	140 192.3	1841.1	139 929.5	1837.6	139 666.7	1834.2
2.3.....	147 113	2020	146 838	2016	146 563	2012	146 288	2008	146 014	2005
2.4.....	153 507	2199	153 220	2195	152 934	2191	152 647	2187	152 360	2183
2.5.....	159 901	2386	159 603	2382	159 304	2377	159 005	2373	158 707	2368
2.6.....	166 295	2581	165 985	2576	165 674	2571	165 364	2567	165 053	2562
2.7.....	172 689	2783	172 367	2778	172 044	2773	171 722	2768	171 399	2763
2.8.....	179 083	2993	178 748	2988	178 414	2982	178 079	2977	177 745	2971
2.9.....	185 476	3211	185 130	3205	184 783	3199	184 437	3193	184 090	3187
3.0.....	191 869	3436	191 511	3430	191 152	3423	190 794	3417	190 436	3411
3.1.....	198 262	3669	197 892	3662	197 521	3655	197 151	3648	196 781	3642
3.2.....	204 654	3910	204 272	3902	203 890	3895	203 508	3888	203 125	3880
3.3.....	211 047	4158	210 652	4150	210 258	4142	209 864	4134	209 470	4127
3.4.....	217 439	4413	217 032	4405	216 626	4397	216 220	4389	215 814	4380
3.5.....	223 830	4677	223 412	4668	222 994	4659	222 576	4651	222 158	4642
3.6.....	230 221	4948	229 791	4939	229 361	4929	228 931	4920	228 501	4911
3.7.....	236 612	5227	236 170	5217	235 728	5207	235 286	5197	234 845	5187
3.8.....	243 003	5513	242 549	5503	242 095	5492	241 641	5482	241 187	5472
3.9.....	249 393	5807	248 927	5796	248 462	5785	247 996	5774	247 530	5763
4.0.....	255 783	6108	255 305	6097	254 828	6085	254 350	6074	253 872	6063
4.1.....	262 172	6417	261 683	6405	261 193	6393	260 704	6381	260 214	6369
4.2.....	268 562	6734	268 060	6722	267 558	6709	267 057	6696	266 555	6684
4.3.....	274 950	7059	274 437	7045	273 923	7032	273 410	7019	272 896	7006
4.4.....	281 339	7391	280 813	7377	280 288	7363	279 763	7349	279 237	7335
4.5.....	287 727	7730	287 189	7716	286 652	7701	286 115	7687	285 577	7673

TABLE III—Continued.

Longitude	Latitude									
	56.5°		56.6°		56.7°		56.8°		56.9°	
	x	*y*	*x*	*y*	*x*	*y*	*x*	*y*	*x*	*y*
Grades	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*
0.02...	1267.4	0.2	1265.1	0.2	1262.7	0.2	1260.3	0.2	1257.9	0.2
0.04...	2534.9	0.6	2530.1	0.6	2525.3	0.6	2520.6	0.6	2515.8	0.6
0.06...	3802.4	1.4	3795.2	1.4	3788.0	1.4	3780.8	1.4	3773.7	1.4
0.08...	5069.8	2.4	5060.2	2.4	5050.7	2.4	5041.1	2.4	5031.6	2.4
0.10...	6337.3	3.8	6325.3	3.8	6313.4	3.8	6301.4	3.8	6289.4	3.8
0.12...	7604.7	5.4	7590.4	5.4	7576.0	5.4	7561.7	5.4	7547.3	5.4
0.14...	8872.2	7.4	8855.4	7.4	8838.7	7.4	8822.0	7.4	8805.2	7.4
0.16...	10 139.6	9.7	10 120.5	9.6	10 101.4	9.6	10 082.2	9.6	10 063.1	9.6
0.18...	11 407.1	12.3	11 385.5	12.2	11 364.0	12.2	11 342.5	12.2	11 321.0	12.2
0.2....	12 674.5	15.1	12 650.6	15.1	12 626.7	15.1	12 602.8	15.0	12 578.9	15.0
0.3....	19 011.8	34.0	18 975.9	34.0	18 940.0	33.9	18 904.2	33.8	18 868.3	33.8
0.4....	25 349.0	60.5	25 301.2	60.4	25 253.4	60.3	25 205.5	60.2	25 157.7	60.1
0.5....	31 686.1	94.6	31 626.4	94.4	31 566.6	94.2	31 506.8	94.0	31 447.1	93.8
0.6....	38 023.3	136.2	37 951.5	135.9	37 879.8	135.7	37 808.1	135.4	37 736.4	135.2
0.7....	44 360.3	185.4	44 276.7	185.0	44 193.0	184.6	44 109.4	184.3	44 025.7	184.0
0.8....	50 697.4	242.1	50 601.7	241.6	50 506.1	241.2	50 410.5	240.7	50 314.9	240.3
0.9....	57 034.3	306.4	56 926.7	305.8	56 819.2	305.2	56 711.6	304.7	56 604.0	304.1
1.0....	63 371.1	378.3	63 251.6	377.6	63 132.1	376.8	63 012.6	376.1	62 893.1	375.4
1.1....	69.707.9	457.7	69 576.4	456.8	69 445.0	456.0	69 313.5	455.1	69 182.0	454.2
1.2....	76 044.6	544.7	75 901.2	543.7	75 757.8	542.6	75 614.3	541.6	75 470.9	540.6
1.3....	82 381.1	639.3	82 225.7	638.1	82 070.4	636.9	81 915.0	635.6	81 759.6	634.4
1.4....	88 717.6	741.4	88 550.3	740.0	88 382.9	738.6	88 215.6	737.2	88 048.3	735.8
1.5....	95 053.9	851.1	94 874.6	849.5	94 695.3	847.9	94 516.1	846.3	94 336.8	844.7
1.6....	101 390.1	968.4	101 198.9	966.5	101 007.6	964.7	100 816.4	962.9	100 625.2	961.0
1.7....	107 726.1	1093.2	107 522.9	1091.1	107 319.7	1089.0	107 116.6	1087.0	106 913.4	1084.9
1.8....	114 062.0	1225.6	113 846.9	1223.2	113 631.8	1220.9	113 416.6	1218.6	113 201.5	1216.3
1.9....	120 397.7	1365.5	120 170.6	1362.9	119 943.6	1360.4	119 716.5	1357.8	119 489.4	1355.2
2.0....	126 733.3	1513.0	126 494.2	1510.2	126 255.2	1507.3	126 016.2	1504.5	125 777.2	1501.6
2.1....	133 068.6	1668.1	132.817.7	1665.0	132 566.7	1661.8	132 315.7	1658.7	132 064.8	1655.5
2.2....	139 403.8	1830.7	139 140.9	1827.3	138 878.0	1823.8	138 615.0	1820.4	138 352.1	1816.9
2.3....	145 739	2001	145 464	1997	145 189	1993	144 914	1990	144 639	1986
2.4....	152 074	2179	151 787	2175	151 500	2170	151 213	2166	150 926	2162
2.5....	158 408	2364	158 109	2360	157 811	2355	157 512	2351	157 213	2346
2.6....	164 742	2557	164 432	2552	164 121	2547	163 810	2542	163 500	2538
2.7....	171 076	2757	170 754	2752	170 431	2747	170 108	2742	169 786	2737
2.8....	177 410	2965	177 076	2960	176 741	2954	176 407	2949	176 072	2943
2.9....	183 744	3181	183 397	3175	183 051	3169	182 704	3163	182 358	3157
3.0....	190 077	3404	189 719	3398	189 360	3391	189 002	3385	188 643	3378
3.1....	196 410	3635	196 040	3628	195 670	3621	195 299	3614	194 929	3607
3.2....	202 743	3873	202 361	3866	201 978	3858	201 596	3851	201 214	3844
3.3....	209 076	4119	208 681	4111	208 287	4103	207 893	4096	207 498	4088
3.4....	215 408	4372	215 002	4364	214 595	4356	214 189	4348	213 783	4339
3.5....	221 740	4633	221 322	4624	220 903	4616	220 485	4607	220 067	4598
3.6....	228 071	4902	227 641	4892	227 211	4883	226 781	4874	226 351	4865
3.7....	234 403	5178	233 960	5168	233 518	5158	233 076	5148	232 634	5139
3.8....	240 733	5461	240 279	5451	239 825	5441	239 371	5430	238 917	5420
3.9....	247 064	5752	246 598	5742	246 132	5731	245 666	5720	245 200	5709
4.0....	253 394	6051	252 916	6040	252 438	6028	251 960	6017	251 483	6006
4.1....	259 724	6358	259 234	6346	258 744	6334	258 255	6322	257 765	6310
4.2....	266 054	6671	265 552	6659	265 050	6646	264 548	6634	264 046	6621
4.3....	272 383	6993	271 869	6980	271 355	6966	270 842	6953	270 328	6940
4.4....	278 712	7322	278 186	7308	277 660	7294	277 134	7280	276 609	7266
4.5....	285 040	7658	284 502	7644	283 965	7629	283 427	7614	282 889	7600

TABLE III—Continued.

Longitude	Latitude 57.0ᴳ		57.1ᴳ		57.2ᴳ		57.3ᴳ		57.4ᴳ	
	x	y	x	y	x	y	x	y	x	y
Grades	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*
0.02...	1255.5	0.2	1253.1	0.2	1250.7	0.2	1248.3	0.1	1245.9	0.1
0.04...	2511.0	0.6	2506.2	0.6	2501.4	0.6	2496.7	0.6	2491.9	0.6
0.06...	3766.5	1.4	3759.3	1.3	3752.2	1.3	3745.0	1.3	3737.8	1.3
0.08...	5022.0	2.4	5012.4	2.4	5002.9	2.4	4993.3	2.4	4983.7	2.4
0.10...	6277.5	3.8	6265.6	3.7	6253.6	3.7	6241.6	3.7	6229.7	3.7
0.12...	7533.0	5.4	7518.7	5.4	7504.3	5.4	7490.0	5.4	7475.6	5.4
0.14...	8788.5	7.3	8771.8	7.3	8755.0	7.3	8738.3	7.3	8721.5	7.3
0.16...	10 044.0	9.6	10 024.9	9.6	10 005.8	9.6	9986.6	9.5	9967.5	9.5
0.18...	11 299.5	12.1	11 278.0	12.1	11 256.5	12.1	11 234.9	12.1	11 213.4	12.0
0.2....	12 555.0	15.0	12 531.1	15.0	12 507.2	14.9	12 483.3	14.9	12 459.4	14.9
0.3....	18 832.5	33.7	18 796.6	33.7	18 760.8	33.6	18 724.9	33.5	18 689.0	33.5
0.4....	25 109.9	60.0	25 062.1	59.8	25 014.3	59.7	24 966.5	59.6	24 918.6	59.5
0.5....	31 387.3	93.7	31 327.6	93.5	31 267.8	93.3	31 208.0	93.1	31 148.2	93.0
0.6....	37 664.7	134.9	37 593.0	134.6	37 521.3	134.4	37 449.5	134.1	37 377.8	133.9
0.7....	43 942.0	183.6	43 858.4	183.3	43 774.7	182.9	43 691.0	182.6	43 607.3	182.2
0.8....	50 219.3	239.8	50 123.7	239.4	50 028.0	238.9	49 932.4	238.4	49 836.7	238.0
0.9....	56 496.5	303.5	56 388.9	302.9	56 281.3	302.4	56 173.7	301.8	56 066.1	301.2
1.0....	62 773.6	374.7	62 654.0	374.0	62 534.5	373.3	62 414.9	372.6	62 295.4	371.8
1.1....	69 050.6	453.4	68 919.1	452.5	68 787.6	451.7	68 656.1	450.8	68 524.6	449.9
1.2....	75 327.5	539.6	75 184.1	538.5	75 040.6	537.5	74 897.2	536.5	74 753.7	535.5
1.3....	81 604.2	633.2	81 448.9	632.0	81 293.5	630.8	81 138.1	629.6	80 982.6	628.4
1.4....	87 881.0	734.4	87 713.6	733.0	87 546.3	731.6	87 378.9	730.2	87 211.5	728.8
1.5....	94 157.5	843.1	93 978.2	841.5	93 799.0	839.9	93 619.6	838.2	93 440.3	836.6
1.6....	100 434.0	959.2	100 242.7	957.4	100 051.5	955.6	99 860.2	953.7	99 668.9	951.9
1.7....	106 710.2	1082.9	106 507.0	1080.8	106 303.9	1078.7	106 100.6	1076.7	105 897.3	1074.6
1.8....	112 986.4	1214.0	112 771.3	1211.7	112 556.1	1209.4	112 340.9	1207.1	112 125.7	1204.8
1.9....	119 262.3	1352.6	119 035.3	1350.0	118 808.2	1347.5	118 581.0	1344.9	118 353.8	1342.3
2.0....	125 538.2	1498.8	125 299.1	1495.9	125 060.1	1493.0	124 821.0	1490.2	124 581.8	1487.3
2.1....	131 813.8	1652.4	131 562.8	1649.2	131 311.8	1646.1	131 060.8	1642.9	130 809.6	1639.8
2.2....	138 089.2	1813.5	137 826.3	1810.0	137 563.4	1806.6	137 300.3	1803.1	137 037.3	1799.7
2.3....	144 364	1982	144 090	1978	143 815	1974	143 540	1971	143 265	1967
2.4....	150 639	2158	150 353	2154	150 066	2150	149 779	2146	149 492	2142
2.5....	156 914	2342	156 616	2337	156 317	2333	156 018	2328	155 719	2324
2.6....	163 189	2533	162 878	2528	162 568	2523	162 257	2518	161 946	2514
2.7....	169 463	2731	169 140	2726	168 818	2721	168 495	2716	168 172	2711
2.8....	175 737	2937	175 403	2932	175 068	2926	174 733	2921	174 399	2915
2.9....	182 011	3151	181 665	3145	181 318	3139	180 971	3133	180 625	3127
3.0....	188 285	3372	187 926	3366	187 568	3359	187 209	3353	186 851	3346
3.1....	194 558	3600	194 188	3594	193 817	3587	193 447	3580	193 076	3573
3.2....	200 831	3836	200 449	3829	200 066	3822	199 684	3815	199 301	3807
3.3....	207 104	4080	206 710	4072	206 315	4064	205 921	4057	205 526	4049
3.4....	213 376	4331	212 970	4323	212 564	4314	212 158	4306	211 751	4298
3.5....	219 649	4590	219 230	4581	218 812	4572	218 394	4563	217 975	4555
3.6....	225 920	4855	225 490	4846	225 060	4837	224 630	4828	224 200	4818
3.7....	232 192	5129	231 750	5119	231 308	5109	230 866	5100	230 423	5090
3.8....	238 463	5410	238 009	5400	237 555	5389	237 101	5379	236 647	5369
3.9....	244 734	5698	244 268	5687	243 802	5676	243 336	5666	242 870	5655
4.0....	251 005	5994	250 527	5983	250 049	5971	249 571	5960	249 093	5948
4.1....	257 275	6298	256 785	6286	256 295	6274	255 805	6262	255 315	6250
4.2....	263 545	6608	263 043	6596	262 541	6583	262 039	6571	261 537	6558
4.3....	269 814	6927	269 300	6914	268 787	6900	268 273	6887	267 759	6874
4.4....	276 083	7252	275 558	7239	275 032	7225	274 506	7211	273 980	7197
4.5....	282 352	7586	281 814	7572	281 277	7557	280 739	7543	280 201	7528

TABLE III—Continued.

Longitude	Latitude									
	57.5^G		57.6^G		57.7^G		57.8^G		57.9^G	
	x	y	x	y	x	y	x	y	x	y
Grades	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*	*Meters*
0.02	1243.5	0.1	1241.2	0.1	1238.8	0.1	1236.4	0.1	1234.0	0.1
0.04	2487.1	0.6	2482.3	0.6	2477.5	0.6	2472.7	0.6	2468.0	0.6
0.06	3730.6	1.3	3723.5	1.3	3716.3	1.3	3709.1	1.3	3701.9	1.3
0.08	4974.2	2.4	4964.6	2.4	4955.0	2.4	4945.5	2.4	4935.9	2.4
0.10	6217.7	3.7	6205.8	3.7	6193.8	3.7	6181.8	3.7	6169.9	3.7
0.12	7461.3	5.3	7446.9	5.3	7432.6	5.3	7418.2	5.3	7403.9	5.3
0.14	8704.8	7.3	8688.1	7.3	8671.3	7.2	8654.6	7.2	8637.8	7.2
0.16	9948.3	9.5	9929.2	9.5	9910.1	9.5	9890.9	9.4	9871.8	9.4
0.18	11 191.9	12.0	11 170.4	12.0	11 148.8	12.0	11 127.3	12.0	11 105.8	11.9
0.2	12 435.4	14.8	12 411.5	14.8	12 387.6	14.8	12 363.7	14.8	12 339.8	14.7
0.3	18 653.1	33.4	18 617.3	33.3	18 581.4	33.3	18 545.5	33.2	18 509.6	33.1
0.4	24 870.8	59.4	24 823.0	59.3	24 775.1	59.2	24 727.3	59.0	24 679.5	58.9
0.5	31 088.4	92.8	31 028.6	92.6	30 968.8	92.4	30 909.1	92.2	30 849.3	92.1
0.6	37 306.0	133.6	37 234.3	133.4	37 162.5	133.1	37 090.8	132.8	37 019.0	132.6
0.7	43 523.6	181.9	43 439.9	181.5	43 356.1	181.2	43 272.4	180.8	43 188.7	180.5
0.8	49 741.0	237.5	49 645.4	237.1	49 549.7	236.6	49 454.0	236.2	49 358.4	235.7
0.9	55 958.5	300.6	55 850.8	300.0	55 743.2	299.5	55 635.6	298.9	55 527.9	298.3
1.0	62 175.8	371.1	62 056.2	370.4	61 936.6	369.7	61 817.0	369.0	61 697.4	368.3
1.1	68 393.0	449.1	68 261.5	448.2	68 129.9	447.3	67 998.4	446.5	67 866.8	445.6
1.2	74 610.2	534.4	74 466.7	533.4	74 323.2	532.4	74 179.7	531.3	74 036.1	530.3
1.3	80 827.2	627.2	80 671.7	626.0	80 516.2	624.8	80 360.8	623.6	80 205.3	622.4
1.4	87 044.1	727.4	86 876.7	726.0	86 709.3	724.6	86 541.8	723.2	86 374.4	721.8
1.5	93 260.9	835.0	93 081.5	833.4	92 902.1	831.8	92 722.8	830.2	92 543.4	828.6
1.6	99 477.5	950.1	99 286.2	948.3	99 094.9	946.4	98 903.6	944.6	98 712.2	942.8
1.7	105 694.0	1072.6	105 490.7	1070.5	105 287.5	1068.4	105 084.2	1066.4	104 880.9	1064.3
1.8	111 910.4	1202.4	111 695.2	1200.1	111 479.9	1197.8	111 264.7	1195.5	111 049.5	1193.2
1.9	118 126.6	1339.8	117 899.4	1337.2	117 672.2	1334.6	117 445.0	1332.0	117 217.8	1329.4
2.0	124 342.7	1484.5	124 103.5	1481.6	123 864.4	1478.8	123 625.2	1475.9	123 386.0	1473.1
2.1	130 558.5	1636.6	130 307.4	1633.5	130 056.3	1630.3	129 805.2	1627.2	129 554.1	1624.0
2.2	136 774.2	1796.2	136 511.1	1792.7	136 248.1	1789.3	135 985.0	1785.8	135 721.9	1782.4
2.3	142 990	1963	142 715	1959	142 440	1956	142 165	1952	141 890	1948
2.4	149 205	2138	148 918	2133	148 631	2129	148 344	2125	148 057	2121
2.5	155 420	2319	155 121	2315	154 822	2311	154 523	2306	154 224	2302
2.6	161 635	2509	161 324	2504	161 013	2499	160 702	2494	160 391	2489
2.7	167 849	2705	167 527	2700	167 204	2695	166 881	2690	166 558	2685
2.8	174 064	2909	173 729	2904	173 394	2898	173 060	2893	172 725	2887
2.9	180 278	3121	179 931	3115	179 584	3109	179 238	3103	178 891	3097
3.0	186 492	3340	186 133	3333	185 774	3327	185 416	3321	185 057	3314
3.1	192 705	3566	192 335	3559	191 964	3553	191 594	3546	191 223	3539
3.2	198 919	3800	198 536	3793	198 154	3785	197 771	3778	197 388	3771
3.3	205 132	4041	204 737	4033	204 343	4026	203 948	4018	203 554	4010
3.4	211 345	4290	210 938	4282	210 532	4273	210 125	4265	209 719	4257
3.5	217 557	4546	217 138	4537	216 720	4528	216 302	4520	215 883	4511
3.6	223 769	4809	223 339	4800	222 908	4791	222 478	4781	222 048	4772
3.7	229 981	5080	229 539	5070	229 096	5061	228 654	5051	228 212	5041
3.8	236 192	5358	235 738	5348	235 284	5338	234 830	5327	234 375	5317
3.9	242 404	5644	241 937	5633	241 471	5622	241 005	5611	240 539	5601
4.0	248 614	5937	248 136	5926	247 658	5914	247 180	5903	246 702	5891
4.1	254 825	6238	254 335	6226	253 845	6214	253 355	6202	252 864	6190
4.2	261 035	6546	260 533	6533	260 031	6520	259 529	6508	259 027	6495
4.3	267 245	6861	266 731	6848	266 217	6834	265 703	6821	265 189	6808
4.4	273 454	7184	272 928	7170	272 402	7156	271 876	7142	271 350	7128
4.5	279 663	7514	279 125	7499	278 587	7485	278 049	7470	277 511	7456

TABLE III—Continued.

Longitude	Latitude, 58.0G		Longitude	Latitude, 58.0G		Longitude	Latitude, 58.0G	
	x	y		x	y		x	y
Grades	*Meters*	*Meters*	*Grades*	*Meters*	*Meters*	*Grades*	*Meters*	*Meters*
0.02	1231.6	0.1	1.0	61 577.8	367.6	3.0	184 698	3308
0.04	2463.2	0.6	1.1	67 735.3	444.8	3.1	190 852	3532
0.06	3694.8	1.3	1.2	73 892.6	529.3	3.2	197 006	3763
0.08	4926.3	2.4	1.3	80 049.9	621.2	3.3	203 159	4002
			1.4	86 207.0	720.4	3.4	209 312	4249
0.10	6157.9	3.7						
0.12	7389.5	5.3	1.5	92 364.0	827.0	3.5	215 465	4502
0.14	8621.1	7.2	1.6	98 520.9	940.9	3.6	221 617	4763
0.16	9852.7	9.4	1.7	104 677.6	1062.2	3.7	227 769	5031
0.18	11 084.3	11.9	1.8	110 834.2	1190.9	3.8	233 921	5307
			1.9	116 990.6	1326.9	3.9	240 072	5590
0.2	12 315.8	14.7						
0.3	18 473.7	33.1	2.0	123 146.9	1470.2	4.0	246 224	5880
0.4	24 631.6	58.8	2.1	129 303.0	1620.9	4.1	252 374	6178
			2.2	135 458.8	1778.9	4.2	258 525	6483
0.5	30 789.5	91.9	2.3	141 615	1944	4.3	264 675	6795
0.6	36 947.3	132.3	2.4	147 770	2117	4.4	270 824	7114
0.7	43 105.0	180.1						
0.8	49 262.7	235.2	2.5	153 925	2297	4.5	276 974	7441
0.9	55 420.3	297.7	2.6	160 080	2485			
			2.7	166 235	2679			
			2.8	172 390	2881			
			2.9	178 544	3091			

MATHEMATICAL DEVELOPMENT OF THE RIGID FORMULA FOR LAMBERT'S PROJECTION.*

If a curved surface is represented by paremetric equations in terms of two variables, u and v, in such a way that the element of length upon the surface becomes

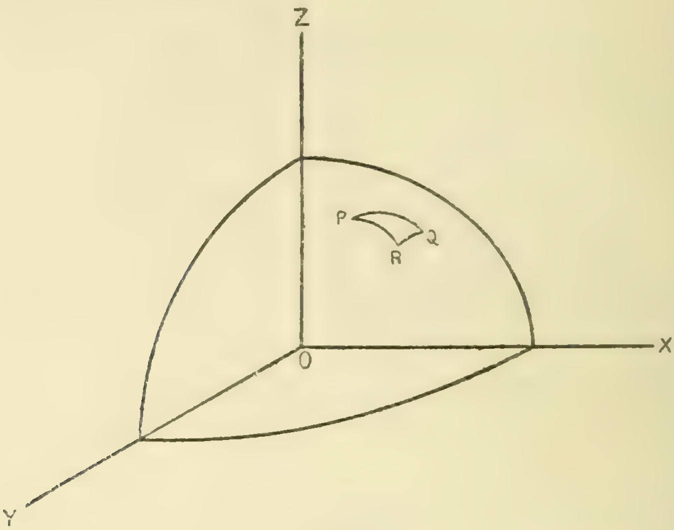

FIG. 7.

$$\overline{PQ}^2 = dS^2 = M^2\ (du^2 + dv^2)\ ,$$

the surface can be represented upon a plane so as to preserve the similarity of infinitesimal elements. The quantity M may be a constant or a function of u and v, but must be independent of differentials.

Let the element in the plane corresponding to dS be

$$\overline{pq}^2 = ds^2 = dx^2 + dy^2 .$$

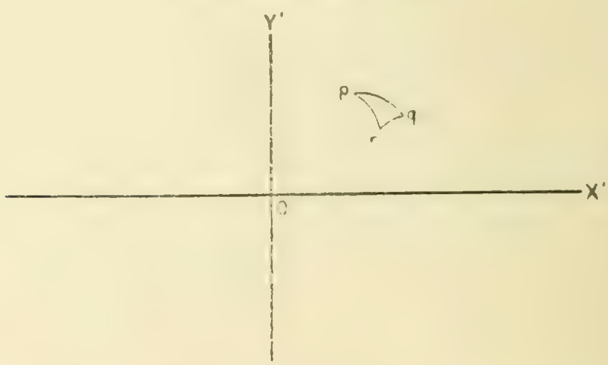

FIG. 8.

* By Oscar S. Adams, Coast and Geodetic Survey.

If another curve upon the surface starts at the same point P, the element of this curve may be represented by

$$\overline{PR}^2 = dS_1^{\,2} = M^2 \,(du_1^{\,2} + dv_1^{\,2}),$$

M being a constant at the point P. The corresponding element in the plane is

$$\overline{pr}^2 = ds_1^{\,2} = dx_1^{\,2} + dy_1^{\,2}.$$

If x is set equal to u and y to v, the relation becomes

$$\frac{dS^2}{ds^2} = M^2 = \frac{dS_1^{\,2}}{ds_1^{\,2}}$$

since M is constant for the point P, and the same for both elements of arc.

This gives

$$\frac{dS}{dS_1} = \frac{ds}{ds_1}.$$

If dS and dS_1 are referred to axes in the tangent plane at P with the point P as origin, we may write

$$dS = \sigma e^{i\theta}$$

$$dS_1 = \sigma' e^{i\theta'}$$

σ and σ' being the lengths of the elements and θ and θ' the angles that they make with the initial line and i denoting as usual $\sqrt{-1}$.

In like manner with p as origin,

$$ds = \eta e^{i\phi}$$

$$ds_1 = \eta' e^{i\phi'}$$

The proportion now becomes

$$\frac{\sigma e^{i\theta}}{\sigma' e^{i\theta'}} = \frac{\eta e^{i\phi}}{\eta' e^{i\phi'}}$$

or,
$$\frac{\sigma}{\sigma'} e^{i(\theta-\theta')} = \frac{\eta}{\eta'} e^{i(\phi-\phi')}.$$

Hence,
$$\frac{\sigma}{\sigma'} = \frac{\eta}{\eta'},$$

and
$$\theta - \theta' = \phi - \phi'.$$

Therefore the elementary triangle PQR is similar to the elementary triangle pqr, having two sides of the one proportional to two sides

of the other and the included angles equal. This establishes the similarity of elementary parts of the surface and the plane. This is called by Gauss conformal representation. It is also called the orthomorphic projection. After the curved surface is mapped in this manner upon the plane, this plane can be conformally mapped upon another plane by setting

$$x+iy=f\ (u+iv),$$

the symbol f denoting an arbitrary function.

By differentiation,

$$dx+idy=f'\ (u+iv)\ (du+idv).$$

Also,
$$dx-idy=f'\ (u-iv)\ (du-idv).$$

By multiplication,

$$dx^2+dy^2=f'\ (u+iv)\,f'\ (u-iv)\ (du^2+dv^2)$$

or,
$$ds_2^{\,2}=m^2dS^2$$

in which

$$m^2M^2=f'\ (u+iv)\,f'\ (u-iv).$$

Thus the second plane is a conformal representation of the original surface as well as of the first plane. The same thing will be true if any one of the following relations is used:

$$x-iy=f_1\ (u+iv)$$

$$x+iy=f_2\ (u-iv)$$

or,
$$x-iy=f_3\ (u-iv).$$

These are the general solutions of the problem given by Gauss.

If the surface to be represented is an ellipsoid of revolution, the parametric equations may be used in the following form:

$$x=a\ \cos\ M\sin\ u$$

$$y=a\ \sin\ M\sin\ u$$

$$z=b\ \cos\ u$$

a is the semimajor axis, b is the semiminor axis, M is the longitude, and u the eccentric angle of the generating ellipse, or the complement of the reduced latitude.

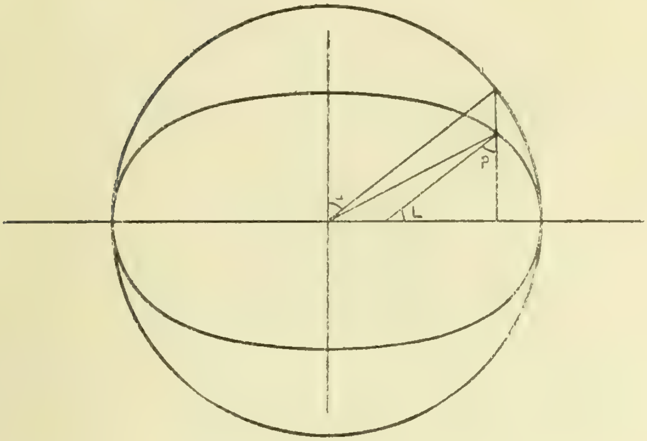

FIG. 9.

The element of length upon the spheroid becomes

$$ds^2 = dx^2 + dy^2 + dz^2.$$

But,
$$dx = a \cos M \cos u \, du - a \sin M \sin u \, \overline{dM}$$
$$dy = a \sin M \cos u \, du + a \cos M \sin u \, \overline{dM}$$
$$dz = -b \sin u \, du,$$

hence,
$$ds^2 = a^2 \sin^2 u \, \overline{dM}^2 + (a^2 \cos^2 u + b^2 \sin^2 u) \, du^2.$$

If $\frac{a^2 - b^2}{a^2}$ is put equal to ϵ^2, this equation becomes

$$ds^2 = a^2 \sin^2 u \left[\overline{dM}^2 + (\cot^2 u + 1 - \epsilon^2) \, du^2 \right].$$

If p is the complement of the latitude L, the relation between p and u is

$$\frac{b}{a} \tan u = \tan p$$

or
$$\sqrt{1 - \epsilon^2} \tan u = \tan p$$

$$\cos^2 u = \frac{(1 - \epsilon^2) \cos^2 p}{1 - \epsilon^2 \cos^2 p}.$$

$$\left(\text{From the relation } \cos^2 u = \frac{1}{1 + \tan^2 u} \right)$$

$$\sin^2 u = \frac{\sin^2 p}{1 - \epsilon^2 \cos^2 p}$$

$$\frac{\sqrt{1 - \epsilon^2} \, du}{\cos^2 u} = \frac{dp}{\cos^2 p}$$

$$du = \frac{\sqrt{1 - \epsilon^2} \, dp}{1 - \epsilon^2 \cos^2 p}$$

$$(\cot^2 u + 1 - \epsilon^2)\, du^2 = \frac{(1-\epsilon^2)^2\, dp^2}{(1-\epsilon^2 \cos^2 p)^2 \sin^2 p}$$

hence,

$$dS^2 = \frac{a^2 \sin^2 p}{1-\epsilon^2 \cos^2 p}\left[\overline{dM}^2 + \frac{(1-\epsilon^2)^2\, dp^2}{(1-\epsilon^2\cos^2 p)^2 \sin^2 p}\right]$$

Let,

$$d\theta = \frac{(1-\epsilon^2)\, dp}{(1-\epsilon^2 \cos^2 p) \sin p}$$

then,

$$\theta = \int \frac{dp}{\sin p} - \frac{\epsilon}{2}\int \frac{\epsilon \sin p\, dp}{1-\epsilon \cos p} + \frac{\epsilon}{2}\int \frac{-\epsilon \sin p\, dp}{1+\epsilon \cos p}$$

$$\theta = \log \tan \frac{p}{2} - \frac{\epsilon}{2} \log (1-\epsilon \cos p) + \frac{\epsilon}{2} \log (1+\epsilon \cos p) + \log G$$

$$\theta = \log\left[G \tan \frac{p}{2} \cdot \left(\frac{1+\epsilon \cos p}{1-\epsilon \cos p}\right)^{\frac{\epsilon}{2}}\right]$$

G being a constant of integration which by a proper choice of limits may be made equal to unity. This gives

$$dS^2 = \frac{a^2 \sin^2 p}{1-\epsilon^2 \cos^2 p}(\overline{dM}^2 + d\theta^2)$$

As has been shown by the theory of functions of a complex variable the spheroid may be conformally mapped upon a plane by letting

$$x + iy = f\,(M \pm i\theta)$$

f denoting an arbitrary function. If

$$f\,(v) = kv,$$

by using the lower sign it is found that

$$x + iy = kM - ik \log \tan \frac{p}{2} \cdot \left(\frac{1+\epsilon \cos p}{1-\epsilon \cos p}\right)^{\frac{\epsilon}{2}}$$

By equating the real parts and the imaginary parts, there result

$$x = kM$$

$$y = k \log\left[\cot \frac{p}{2} \cdot \left(\frac{1-\epsilon \cos p}{1+\epsilon \cos p}\right)^{\frac{\epsilon}{2}}\right]$$

p being the complement of the latitude and the log sign denoting the Naperian logarithm. This is the Mercator projection for the spheroid.

If,

$$f\,(v) = Ke^{ilv},$$

with the lower sign again, the equations become

$$x + iy = Ke^{ilM + \log\left[\tan\frac{p}{2}\cdot\left(\frac{1+\epsilon\cos p}{1-\epsilon\cos p}\right)^{\frac{\epsilon}{2}}\right]^{l}}$$

$$x + iy = K \tan^l \frac{p}{2} \cdot \left(\frac{1+\epsilon \cos p}{1-\epsilon \cos p}\right)^{\frac{l\epsilon}{2}} (\cos lM + i \sin lM)$$

On equating the real parts and the imaginary parts,

$$x = K \tan^l \frac{p}{2} \cdot \left(\frac{1+\epsilon \cos p}{1-\epsilon \cos p}\right)^{\frac{l\epsilon}{2}} \cos lM$$

$$y = K \tan^l \frac{p}{2} \cdot \left(\frac{1+\epsilon \cos p}{1-\epsilon \cos p}\right)^{\frac{l\epsilon}{2}} \sin lM$$

This projection makes the parallels concentric circles of radius

$$r = K \tan^l \frac{p}{2} \cdot \left(\frac{1+\epsilon \cos p}{1-\epsilon \cos p}\right)^{\frac{l\epsilon}{2}}$$

The meridians become radii of these concentric circles. This method of mapping is called Lambert's conformal conic projection.

If an angle z is assumed such that,

(1) $$\tan \frac{z}{2} = \tan \frac{p}{2} \cdot \left(\frac{1+\epsilon \cos p}{1-\epsilon \cos p}\right)^{\frac{\epsilon}{2}}$$

this angle z is very nearly equal to the complement of the geocentric latitude.

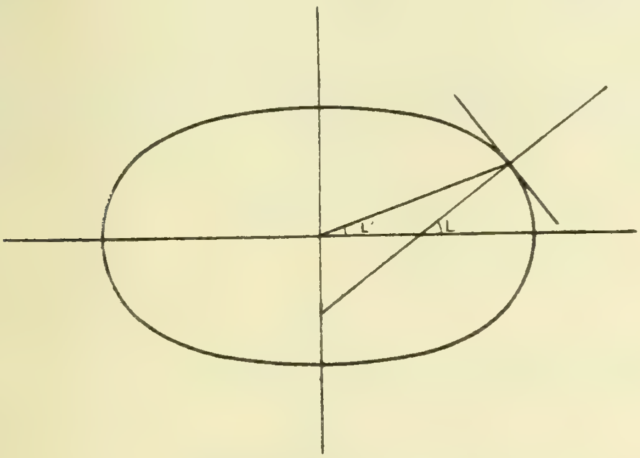

FIG. 10.

If L' is the geocentric latitude and L the geodetic latitude (see fig. 10), then,

(2) $$\tan L' = \frac{b^2}{a^2} \tan L$$

a being the semimajor axis and b the semiminor axis of the spheroid. Then to a sufficent degree of approximation,

(3) $$z = 90° - L'.$$

The equations now become

$$x = K \tan^l \frac{z}{2} \cos lM$$

$$y = K \tan^l \frac{z}{2} \sin lM$$

$$r = K \tan^l \frac{z}{2}$$

With these values x is reckoned downward from the center of the concentric circles and y to the right of the central meridian if M is reckoned positive in that direction. This leaves K and l arbitrary constants. l may be so determined that the ratio of the lengths of two arcs on the map that represent given arcs on the parallels may be in the same ratio as the arcs upon the earth.

If N is the radius of curvature perpendicular to the meridian or the length of the normal to the minor axis, a radian of the parallel of L_1 has the length $N_1 \cos L_1$; of parallel L_2 the length is likewise $N_2 \cos L_2$. Hence the ratio of the lengths is represented by $\frac{N_1 \cos L_1}{N_2 \cos L_2}$. Since the A factor in the tables for the computation of geodetic positions* is equal to $\frac{1}{N \sin 1''}$, this ratio becomes $\frac{A_2 \cos L_1}{A_1 \cos L_2}$.

The arc upon the map that represents the radian of parallel L_1 has the length

$$lr_1 = lK \tan^l \frac{z_1}{2}$$

so, also, that for parallel L_2

$$lr_2 = lK \tan^l \frac{z_2}{2}$$

The ratio of lengths will be preserved if

$$\left(\frac{\tan \frac{z_1}{2}}{\tan \frac{z_2}{2}} \right)^l = \frac{A_2 \cos L_1}{A_1 \cos L_2}$$

or,

$$(4) \qquad l = \frac{\log \cos L_1 - \log \cos L_2 - \log A_1 + \log A_2}{\log \tan \frac{z_1}{2} - \log \tan \frac{z_2}{2}}$$

K may now be determined so as to hold not only the ratio of the arcs of parallels L_1 and L_2 but also to hold the exact length of these

* See United States Coast and Geodetic Survey Special Publication No. 8.

parallels. This is an excellent determination for the mapping of an area such as that of the United States. This would give

$$lK \tan^{l} \frac{z_1}{2} = N_1 \cos L_1 = \frac{\cos L_1}{A_1 \sin 1''}$$

Hence,

$$(5)\quad K = \frac{\cos L_1}{A_1 \sin 1'' \, l \tan^{l} \frac{z_1}{2}} = \frac{\cos L_2}{A_2 \sin 1'' \, l \tan^{l} \frac{z_2}{2}}$$

If the parallels to be held are chosen about one-sixth of the distance from the bottom and the top of the area to be mapped, the proper balance will be preserved. The top and bottom of the map will then be about as much too large in scale as the central part is too small. The scale along L_1 and L_2 will be exactly correct. With this value of K one can tell how much any parallel is off in scale by computing a radian of the parallel and the length of the arc which represents it. With this projection a map could be made of an area such as that of the United States so that it would not be in error of scale in any part of it by more than $1\frac{1}{2}$ per cent. A polyconic projection of the same area is in error of scale as much as $6\frac{1}{2}$ per cent in some parts. A Lambert projection for the United States should hold the correct scale on parallels 29° and 45°.

If coordinates are to be computed for the mapping of the parallels with origin at the place where the parallel crosses the central meridian, the formulas for computation are then

(6) $\quad r = K \tan^{l} \dfrac{z}{2}$ for the corresponding latitude L

(7) $\quad x = r \sin lM$

(8) $\quad y = 2r \sin^2 \dfrac{lM}{2}$

or $\quad y = x \tan \dfrac{lM}{2}.$

The difference of the radii gives the spacing upon the central meridian. If the top and bottom parallels are constructed by determining the coordinates of the intersections with the meridians, the meridians can then be drawn and each of them subdivided as was done in the case of the central meridian. This will give the coordinates of the other parallels without computation.

If l is set equal to unity, the original equations become

$$x = K \tan \frac{z}{2} \cos M,$$

$$y = K \tan \frac{z}{2} \sin M.$$

This gives a projection for the spheroid analogous to the stereographic projection for the sphere. If the sphere is considered, the value of ϵ is zero and the angle z becomes the polar distance. The south pole is the pole of projection, and the plane upon which the projection is made is tangent at the north pole.

In the calculation of the elements of a projection on the Lambert rigid formula as deduced on pages 39 and 41, equations (2), (3), (6), (7), and (8), the following nomenclature is observed:

L_1, L_2 are parallels of latitude chosen for the computation; K and l are constants depending on the latitudes L_1, L_2.

L_1', L_2' are geocentric latitudes corresponding to L_1 and L_2.

z_1, z_2 are complements of the geocentric latitudes L_1', L_2'.

r_1, r_2 are radii in meters of the circles representing the parallels of latitude L_1, L_2.

(9) x and y are coordinates in meters, for mapping the parallel, the origin being at the point where the parallel crosses the central meridian.

(10) M is distance in degrees of point x, y, from the central meridian measured along the parallel.

(11) $\theta = lM$ is the angle of convergence of the meridian drawn through the point x, y.

log A_1, log A_2 are factors corresponding to L_1, L_2 from United States Coast and Geodetic Survey Special Publication No. 8.

APPLICATION OF THE RIGID FORMULA OF LAMBERT.

1. **For a Map of Northeastern France** in the same geographic area as that covered by the French approximate formula, the standard parallels chosen are the same:

$$L_1 = 47°\ 42'\ (53^G)$$

$$L_2 = 51°\ 18'\ (57^G)$$

NOTE.—In the case of the French approximate formula, the projection was based on these parallels, but the x and y coordinates were not computed for them, the projection being constructed on even degrees. The sample computations are merely given to illustrate the application of the rigid formula.

From (1) and (2), page 39, we have

$$\tan L_1' = \frac{b^2}{a^2} \tan L_1*$$

$$\log \tan L_1' = 2 \log b - 2 \log a + \log \tan 47°\ 42'$$

$$L_1' = 47°\ 30'\ 22''.4$$

From (3), page 39, $z_1 = 90° - L_1'$

Hence,

$$z_1 = 42°\ 29'\ 37''.6$$

$$\frac{z_1}{2} = 21°\ 14'\ 48''.8$$

In like manner,

$$L_2' = 51°\ 06'\ 36''.0$$

$$z_2 = 38°\ 53'\ 24''.0$$

$$\frac{z_2}{2} = 19°\ 26'\ 42''.0$$

From (4), page 40, we have

$$l = \frac{\log \cos L_1 - \log \cos L_2 - \log A_1 + \log A_2}{\log \tan \frac{z_1}{2} - \log \tan \frac{z_2}{2}}$$

*a = equatorial semiaxis = 6 378 206 meters.
b = polar semiaxis = 6 356 584 meters.

Substituting the values of the functions of L_1, L_2, $\frac{z_1}{2}$, $\frac{z_2}{2}$, and taking $\log A_1$ and $\log A_2$ corresponding to L_1, L_2 from the tables (Special Publication No. 8), we have

$$l = 0.760528, \quad \log l = 9.8811152 - 10$$

From (5), page 41, we have

$$K = \frac{\cos L_1}{A_1 \sin 1'' \, l \tan^l \frac{z_1}{2}} = \frac{\cos L_2}{A_2 \sin 1'' \, l \tan^l \frac{z_2}{2}}$$

or,

$$\log K = \log \cos L_1 - \left(\log A_1 + \log \sin 1'' + \log l + l \log \tan \frac{z_1}{2} \right)$$

$$\begin{array}{ll}
\log A_1 & = 8.5089210 - 10 \\
\log \sin 1'' & = 4.6855749 - 10 \\
\log l & = 9.8811152 - 10 \\
{}^{*}l \log \tan \frac{z_1}{2} & = 9.6879891 - 10 \\ \hline
 & 2.7636002 - 10 \\
\log \cos L_1 & = 9.8280231 - 10 \\
\log K & = 7.0644229
\end{array}$$

Solving for K with $L_2 = 51° \; 18'$ gives the same value.

From (6), page 41, we have

$$\begin{array}{ll}
r_1 & = K \tan^l \frac{z_1}{2} \\
\log r_1 & = \log K + l \log \tan \frac{z_1}{2} \\
\log K & = 7.0644229 \\
l \log \tan \frac{z_1}{2} & = 9.6879891 - 10 \\ \hline
\log r_1 & = 6.7524120
\end{array}$$

$r_1 = 5654732$ meters for parallel of latitude L_1 $(47° \; 42')$

---

* $\log \tan \frac{z_1}{2} = 9.5897444 - 10$, $l = 0.760528$; $l \log \tan \frac{z_1}{2} = 7.2932691 - 7.60528$

Add and subtract 2.39472 from last term,— $\quad +2.39472 \quad -2.39472$

$l \log \tan \frac{z_1}{2} = 9.6879891 - 10$

To compute the coordinates for mapping the parallel 47° 42′. (See (7) and (8), p. 41, and (9), p. 42.)

$$x = r_1 \sin lM$$

$$y = 2r_1 \sin^2 \frac{lM}{2}$$

Take

$M = 7°$. (See (10), p. 42.)

Then

$\theta = lM = 7° \times .760528 = 5°\ 19'\ 25''.3$. (See (11), p. 42.)

Then

$x = r_1 \sin \theta = 5654732 \times \sin\ 5°\ 19'\ 25''.3 = 524659.3$ meters.

$y = 2r_1\ \sin^2 \frac{\theta}{2} = 2 \times 5654732 \times \sin^2 2°\ 39'\ 42''.65 = 24392.2$ meters.

This gives the coordinates of intersection of parallel 47° 42′ with the meridian 7° distant in longitude from the central meridian. By choosing corresponding values of M, the coordinates of intersection of other meridians with the parallel may be computed.

In the same manner compute the value of r_2 for latitude $L_2 = 51°\ 18'$, using the same values of K and l

$r_2 = 5254471$ meters.

For

$M = 7°$, $lM = 5°\ 19'\ 25''.3$,

$x = 487522.2$ meters,

$y = 22665.6$ meters.

Other coordinates for the intersection of parallel 51° 18′ with the meridians may be computed by taking the desired values of M. With the coordinates of intersection of the meridians with the top and bottom parallels computed and mapped, the other parallels may be obtained by subdivision, or the proper spacings may be determined by computing the radii for the desired parallels.

2. For a map of the United States, the middle parallel is 37° and the limits in latitude 25° and 49°. The parallels chosen for computation are 29°* and 45°.

Whence, $L_1 = 29°$, $L_2 = 45°$.

* By assuming parallels 31° and 45° as standards, the scale error in the central part of the United States could be reduced, while the scale error would be increased for only a small portion of southern Florida and Texas.

Computing the values of the different quantities as expressed by the equations we have

$L'_1 = 28° 50' 07''.0$

$z_1 = 61° 09' 53''.0$

$\frac{z_1}{2} = 30° 34' 56''.5$

$L'_2 = 44° 48' 19''.6$

$z_2 = 45° 11' 40''.4$

$\frac{z_2}{2} = 22° 35' 50''.2$

$\log l = 9.7809125 - 10$

$l = 0.603827$

$\log K = 7.1038803$

For latitude 29° (L_1)

$r_1 = 9245940$ meters.

Take

$M = 1°$, $\theta = lM = (.603827)\ (1°) = 36'\ 13''.8$,

$x = 97440.0$ meters,

$y = 513.5$ meters.

For latitude 45° (L_2),

$r_2 = 7481820$ meters.

For

$M = 1°$, $\theta = lM = 36'\ 13''.8$,

$x = 78848.6$ meters,

$y = 415.5$ meters.

SYSTEM OF KILOMETRIC SQUARES USED ON LAMBERT'S PROJECTION IN FRANCE.

[See Plates II and VII.]

In the maps and quadrangled areas of the eastern part of France, the central point chosen for the projection is

$$L_0 = 55^G,\ M_0 = -6^G\ (6^G \text{ east of Paris})$$

This point is found ESE. of Trèves and is the initial point of geographic coordinates. (See Plate II.)

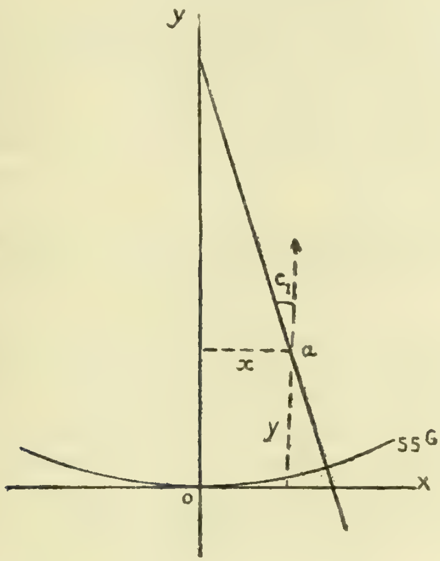

FIG. 11.

The y – axis (fig.11) is the straight line which represents the initial meridian of -6^G, and the x – axis is the tangent to the circle which represents the parallel of 55^G at its point of intersection with the y – axis. The point a of the projection will be defined by x and y.

In order not to have negative values west and south of the central point, the point o is given the coordinates

$$X_0 = 500{,}000 \text{ meters}$$
$$Y_0 = 300{,}000 \text{ meters}$$

The coordinates of a will be

$$X = 500{,}000 + x$$
$$Y = 300{,}000 + y$$

At the point a, the meridian of the place makes an angle with the $y-$axis equal to the convergence of the meridians.

$=(M-M_0)\sin L_0=(M+6^G)\times 0.76$, with sign such that the direction geographic north shall converge toward the initial meridian of -6^G (6^G east of Paris).

EXAMPLE.—The map on the scale 1–80,000 is issued in rectangular sheets 64 kilometers east and west, by 40 kilometers north and south. The vertical border is parallel to the meridian of Paris. What will be the inclination of the y-axis of the system of squares on Lambert's projection to the vertical line of the border?

This angle $c_1=(0+6^G)\times 0.76=4.56^G$ toward the west.

To place the system of kilometric grids on a map or projection that does not have the system printed thereon:

1. Construct a kilometric grid to the proper scale on tracing paper.
2. Place the geographic points (shown in red on Plate VII) by their rectangular coordinates on the grid. Only one such point appears on this plate.
3. Superimpose the tracing paper grid on the map and locate its exact position by the coincidence of geographic points on grid and map.
4. Transfer the grid by pricking points through the boundaries of the squares and complete the system by ruling lines through these points.

On many quadrillages (systems of kilometric squares), the eastern and western neat lines are not parallel to the meridian of Paris, but conform to the meridian which passes through the origin of the system of squares. The true and magnetic north are indicated on the margin and their angular departure from the meridional lines of the kilometric system of squares is expressed in grades and tenths.

On French maps the prime meridian is Paris (2° 20′ 14″ east of Greenwich), and in recent practice the geographic projection has been subdivided into grades which are one-hundredth of a quadrant, or nine-tenths of a degree.

Part 2.—COMPARISON OF THE LAMBERT CONFORMAL CONIC PROJECTION WITH THE BONNE AND POLYCONIC PROJECTIONS.

LAMBERT'S PROJECTION.

[See Plates, I, II, and III.]

This projection is of the simple conical type in which all meridians are straight lines that meet in a common point beyond the limits of the map, and the parallels are concentric circles whose center is at the point of intersection of the meridians. Meridians and parallels

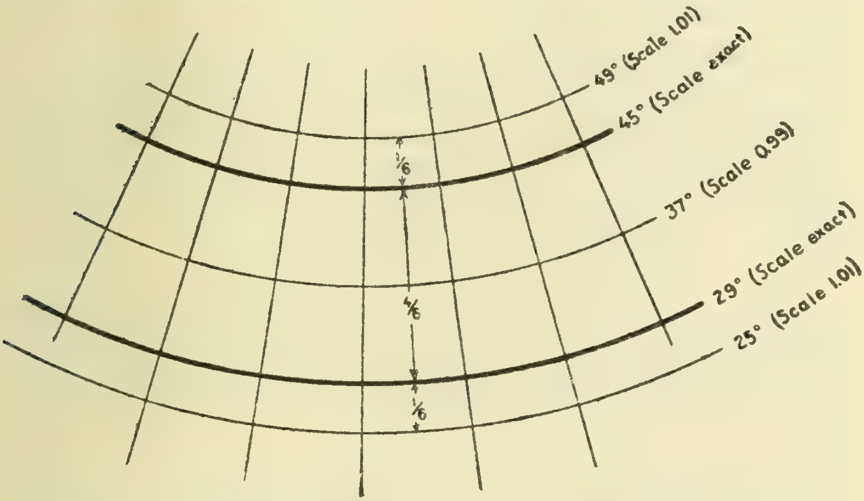

Diagram showing maximum distortion for map of United States as based on Lambert's Conformal Conic Projection.

FIG. 12.

intersect at right angles and the angles formed by any two lines on the earth's surface are correctly represented on this projection.

It employs a cone intersecting the spheroid at two parallels known as the standard parallels for the area to be represented. In general, for equal distribution of scale error, the standard parallels are chosen at one-sixth and five-sixths of the total length of that portion of the central meridian to be represented. It may be advisable in some localities, or for special reasons, to bring them closer together in

order to have greater accuracy in the center of the map at the expense of the upper and lower border areas.

On these two selected parallels, arcs of longitude are represented in their true lengths.

Between these selected parallels the scale will be a trifle too small and beyond them the scale will be too large.

This projection is specially suited for maps having a predominating east and west dimension.

On page 45, the preliminary operations for constructing a map of the United States on this projection are given. The standard parallels selected are 29° and 45°.

The chief advantage of this projection over the polyconic, as used by several Government bureaus for maps of the United States, consists in reducing the scale error along the western border of the United States from 6½ to 1 per cent. It would, also, by the application of this rigid Lambert formula here presented, give us a projection that is exactly conformal.

GENERAL OBSERVATIONS ON THE LAMBERT PROJECTION.

In the construction of the map of France (Pls. I, II, and III), which was extended to 7° of longitude from the middle meridian for purposes of comparison with the polyconic projection of the same area, the following results were noted:

Maximum scale error, Lambert =0.05 per cent.
Maximum scale error, polyconic=0.32 per cent.

Azimuthal and right line tests for orthodrome (great circle) also indicated a preference for the Lambert projection in these two vital properties, these tests indicating accuracies for the Lambert projection well within the errors of map construction and paper distortion.

In respect to areas, in a map of the United States, it should be noted that while in the polyconic projection they are misrepresented along the western margin in one dimension (that is, by meridional distortion of 6½ per cent), on the Lambert projection they are distorted along both the parallel and meridian as we depart from the standard parallels, with a resulting maximum error of 2 per cent.

The maximum error in scale for a map of the United States on a Lambert projection would be about 1 per cent.

In the Lambert projection for the map of France, the maximum scale errors do not exceed 1–2,000 and are practically negligible, while the angles measured on the map made by this system are practically equal to those on the earth.

It should be remembered, however, that in the Lambert conformal conic, as well as all other conic projections, the scale errors vary

increasingly with the range of latitude north or south of the standard parallels.

It follows, then, that this type of projections is not suited for maps having extensive latitudes.

Areas.—For areas, as stated before, the Lambert projection is better than the polyconic for maps like the one of France or for the United States, where we have wide longitude and comparatively narrow latitude. On the other hand, areas are not represented as well in the Lambert projection or in the polyconic projection as they are in the Bonne or in other conical projections.

For the purpose of equivalent areas of large extent the Lambert's zenithal (or azimuthal) equal area projection offers advantages desirable for census or statistical purposes superior to other projections, excepting in areas of wide longitudes combined with narrow latitudes, where Albers' conical equal area projection with two standard parallels, is preferable.

In measuring areas on a map by the use of a planimeter, the distortion of the paper, due to the method of printing and to changes in the humidity of the air, must also be taken into consideration. It is better to disregard the scale of the map and to use the quadrilaterals formed by the latitude and longitude lines as units. The areas of quadrilaterals of the earth's surface are given for different extents of latitude and longitude in the Smithsonian Geographical Tables, 1897, Tables 25 to 29.

It follows, therefore, that for the various purposes a map may be put to, if the property of areas is slightly sacrificed and the several other properties more desired are retained, we can still by judicious use of the planimeter or Geographical Tables overcome this one weaker property.

The idea seems to prevail among many that, while in the polyconic projection every parallel of latitude is developed upon its own cone, the multiplicity of cones so employed necessarily adds strength to the projection; but this is not true.

The ordinary polyconic projection has, in fact, only one line of strength; that is, the central meridian. In this respect then it is no better than the Bonne.

The Lambert projection, on the other hand, employs two lines of strength which are parallels of latitude suitably selected for the region to be mapped.

A line of strength is here used to denote a singular line characterized by the fact that the elements along it are truly represented in shape and scale.

The Lambert, besides this advantage, is adapted to indefinite east and west extensions, a property belonging to this general class of single cone projections, but not found in the polyconic, where ad-

jacent sheets have a "rolling fit" because the meridians are curved in opposite directions.

The question of choice between the Lambert and the polyconic system of projection resolves itself largely into a study of the shapes of the areas involved.

The merits and defects of the Lambert and the polyconic projections may briefly be stated as being, in a general way, in opposite directions.

THE BONNE PROJECTION.

[See Plate IV.]

In this projection a central meridian and a standard parallel are assumed with a cone tangent along the standard parallel. The central meridian is developed along that element of the cone which is tangent to it and the cone developed on a plane.

The standard parallel falls into an arc of a circle with its center at the apex of the developing cone, and the central meridian becomes a right line which is divided to true scale. The parallels are drawn as concentric circles at their true distances apart, and all parallels are divided truly and drawn to scale.

Through the points of division of the parallels the meridians are drawn. The central meridian is a straight line; all others are curves, the curvature increasing with the difference in longitude.

The scale along all meridians, excepting the central, is too great, increasing with the distance from the center, and the meridians become more inclined to the parallels, thereby increasing the distortion. The developed areas preserve a strict equality, in which respect this projection is preferable to the polyconic.

USES.—The Bonne* system of projection, still used to some extent in France, will gradually be discontinued and superseded by the Lambert system.

It is also used in countries like Belgium, Netherlands, and Switzerland. In Stieler's Atlas we find a number of maps with this projection; less extensively so, perhaps, in Stanford. This projection is strictly equal area and this has given it its popularity.

In maps of France having the Bonne projection, the center of projection is found at the intersection of the meridian of Paris and the parallel of latitude 50^G. The border divisions and subdivisions appear in grades, minutes (centesimal), seconds, or tenths of seconds.

LIMITATIONS.—Its distortion, as the difference in longitude increases, is its chief defect. On the map of France the distortion at the edges reaches a value of 18′ for angles, and if extended into Alsace, or western Germany, it would have errors in distances which are inadmissible in calculations. In the present rigorous tests of the military operations these errors became too serious for the purposes to which the map is intended to serve.

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*Tables for this projection were computed by Plessis.

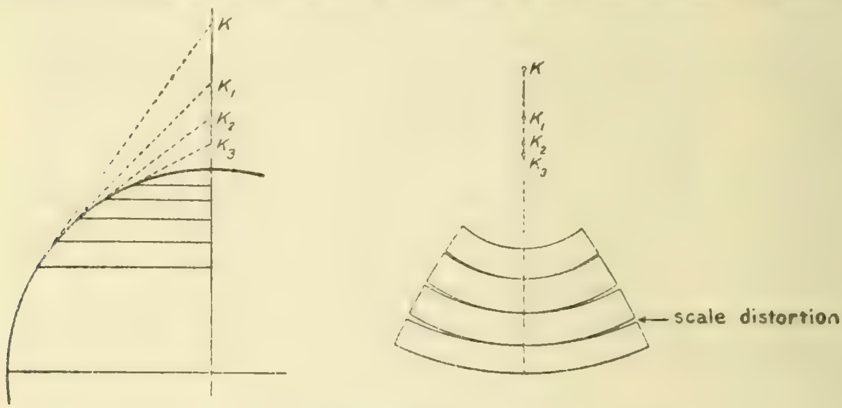
THE POLYCONIC PROJECTION.

[See Plate V.]

The polyconic projection, devised by Ferdinand Hassler, the first Superintendent of the Coast and Geodetic Survey, possesses great popularity on account of mechanical ease of construction and the fact that a general table* for its use has been calculated for the whole spheroid.

It may be interesting to quote Prof. Hassler† in connection with two projections, viz, the intersecting conic projection and the polyconic projection:

1. *Projection on an intersecting cone.*—The projection which I intended to use was the development of a part of the earth's surface upon a cone, either a tangent to a certain latitude, or *cutting two given parallels* and two meridians, equidistant from the middle meridian, and extended on both sides of the meridian, and in latitude,



Polyconic development

FIG. 13.

only so far as to admit no deviation from the real magnitudes, sensible in the detail surveys.

2. *The polyconic projection.*—* * * This distribution of the projection, in an assemblage of sections of surfaces of successive cones, tangents to or cutting a regular succession of parallels, and upon regularly changing central meridians, appeared to me the only one applicable to the coast of the United States.

Its direction, nearly diagonal through meridian and parallel, would not admit any other mode founded upon a single meridian and parallel without great deviations from the actual magnitudes and shape, which would have considerable disadvantages in use.

Figure on left above shows the centers (K, K_1, K_2, K_3) of circles on the projection that represent the corresponding parallels on the

* Tables for the polyconic projection of maps, Coast and Geodetic Survey, Special Publication No. 5.

† Papers on various subjects connected with the survey of the coast of the United States; by F.R. Hassler, communicated March 3, 1820 (in Trans. Am. Phil. Soc., Vol. 2, pp. 405-408, Philadelphia, 1825).

earth. Figure on right above shows the distortion at the outer meridian due to the varying radii of the circles in the polyconic development.

A central meridian is assumed upon which the intersections of the parallels are truly spaced. Each parallel is then separately developed by means of a tangent cone, the centers of the developed arcs of parallels lying in the extension of the central meridian. The arcs of the developed parallels are subdivided to true scale and the meridians drawn through the corresponding subdivisions. Since the radii for the parallels decrease as the cotangent of the latitude, the circles are not concentric and the lengths of the arcs of latitude gradually increase as we recede from the meridian.

The central meridian is a right line; all others are curves, the curvature increasing with the longitudinal distance from the center. The intersections between meridians and parallels also depart from right angles as the distance increases.

From the construction of the projection it is seen that errors in meridional distances, areas, shapes, and intersections increase with the longitudinal limits. It therefore should be restricted in its use to maps of wide latitudes and narrow longitudes.

The *polyconic* projection may be considered as in a measure *only compromising* various conditions impossible to be represented on any one map or chart, such as relate to—

First. Rectangular intersections* of parallels and meridians.

Second. Equal scale* over the whole extent (the error in scale in latitude 39° 00′ not exceeding 1 per cent for distances within 560 statute miles of the great circle used as its central meridian).

Third. Facilities for using great circles and azimuths within distances just mentioned.

Fourth. Proportionality of areas* with those on the sphere, etc.

If the map should have a predominating east and west dimension, the polyconic properties may still be retained, however, by applying the developing cones in a transverse position. A great circle at right angles to a central meridian at the middle part of the map can be made to play the part of the central meridian, the poles being transferred (in construction only) to the Equator. By transformation of coordinates a projection may be completed which will give all poly-

* The errors in meridional scale and area are expressed very closely by the formula

$$E = +0.01 \left(\frac{l^\circ \cos \phi}{8.1}\right)^2$$

in which l° = distance of point from central meridian expressed in degrees of longitude, and ϕ = latitude.

EXAMPLE.—For latitude 39° the error for 10° 27′ (560 statute miles) departure in longitude is 1 per cent for scale along the meridian and the same amount for area.

The angular distortion is a variable quantity not easily expressed by an equation. In latitude 40° this distortion is 1′.1 on the meridian 10° distant from the central meridian; at 30° distant it increases to 28′.7.

The greatest angular distortion in this projection is at about latitude 37°, decreasing to zero as we approach the Equator or the pole.

conic properties in a transverse relation. This process is, however, laborious and has seldom been resorted to.

Since the distance across the United States from north to south is only three-fifths of that from east to west, it follows, then, by the above manipulation that the maximum distortion can be reduced from 6½ to 2½ per cent.

A projection of this type is peculiarly suited to a map covering an important section of the North Pacific Ocean. If a great circle passing through San Francisco and Manila is treated in construction as a central meridian in the ordinary polyconic projection, we can cross the Pacific in a narrow belt so as to include the American and Asiatic coasts with a minimum scale distortion. By transformation of coordinates the meridians and parallels can be constructed so that the projection will present the usual appearance and may be utilized for ordinary purposes.

The configuration of the two continents is such that all the prominent features of America and eastern Asia are conveniently close to this selected axis, viz, Panama, Brito, San Francisco, Straits of Fuca, Unalaska, Kiska, Yokohama, Manila, Hongkong, and Singapore. It would be a typical case of a projection being adapted to the configuration of the locality treated. A map on a transverse polyconic projection as here suggested, while of no special navigational value, would be of interest from a geographic standpoint as exhibiting in their true relations a group of important localities covering a wide expanse.

The polyconic projection is by construction not conformal, neither do the parallels and meridians intersect at right angles, as is the case with all conical or single cone projections, whether these latter are conformal or not.

It is sufficiently close to other types possessing in some respects better properties that its great tabular advantages should generally determine its choice within certain limits.

As stated in Hinks' Map Projection, it is a link between those projections which have some definite scientific value and those generally called conventional, but possess properties of convenience and use.

The three projections herein compared may be considered as practically identical within areas not distant more than 3° from a central point, the errors from construction and distortion of the paper exceeding those due to the system of projection used.

LAMBERT'S ZENITHAL (OR AZIMUTHAL) EQUAL AREA PROJECTION.*

[See Plate VI.]

This is probably the most important of the azimuthal equivalent projections.

In this projection the zenith of the central point of the surface to be represented appears as pole in the center of the map; the azimuth of any point within the surface, as seen from the central point, is the same as that for the corresponding points of the map; and from the same central point, in all directions, equal great circle distances to points on the earth are represented by equal linear distances on the map.

It has the additional property that areas on the projection are proportional to the corresponding areas of the sphere.

Within the limits of the map shown the maximum scale error is but $3\frac{1}{2}$ per cent (polyconic projection has $6\frac{1}{2}$ per cent), while the error in angles (excluding the center of the map where they are true) is at most $1\frac{1}{2}°$.

The center used for this projection (Plate VI) is a point in the eastern part of Kansas, in latitude 38° 00′ and longitude 95° 00′. The geographic center † of the United States is approximately in latitude 39° 50′ and longitude 98° 35′.

Areas being true, this projection is admirably suited for census purposes. It has been employed by the Survey Department, Ministry of Finance, Egypt, for the wall map of Asia.

For a map of the whole of the North Atlantic Ocean, this projection offers advantages superior to others. The somewhat circular configuration of the Atlantic basin is more correctly represented by this system of projection than by any other, and with less scale error.

The projection can be carried from a central point for 30° of arc of great circle with a scale error of but $3\frac{1}{2}$ per cent, and for 40° with a scale error of $6\frac{1}{2}$ per cent.

This projection would be admirably suited to a map covering the whole of North America.

The inconvenience of plotting the transcendental curves of parallels and meridians, the nonintersection of these systems at right

* A projection of this type was constructed in the Coast and Geodetic Survey in 1894, but not published.

† "Geographic center of the United States" is here considered as a point analagous to the center of gravity of a spherical surface equally weighted (per unit area) and of the outline of the country, and hence it may be found by means similar to those employed to find the center of gravity.

angles, and the consequent inconvenience of plotting positions, all tend to make this one of the most difficult projections to construct.

It should be remembered also that in areas *smaller* than the whole United States (areas in which the radius from a center common to both projections is not more than 5°) the two projections, Lambert's zenithal and the polyconic, are so nearly identical that the greater labor involved in constructing the Lambert's zenithal projection would hardly justify its use in preference to the polyconic.

The formulas for this projection are not included in this paper, as they are rather complicated and the projection itself is not well known. The fact that it is one of the several devised by Lambert, whose conformal conic projection is receiving considerable attention of late, is the main reason for its appearance in this connection.

CONCLUSION.

Lambert's conformal conic projection, recently adopted by the French, also used for a map of Russia, the basin of the Mediterranean, as well as for maps of Europe and Australia in Debes' Neuer Hand-atlas, has unquestionably superior merits for maps of extended longitudes. Furthermore, it is conformal, all elements retaining their original forms.

Its meridians and parallels cut at right angles and it belongs to the same general formula as Mercator's and the Stereographic, which have stood the test of time, both being likewise conformal projections.

It is an obvious advantage to the general accuracy of the scale of a map to have two standard parallels of true lengths; that is to say, two axes of strength instead of one. As an additional asset, all meridians are straight lines, as they should be.

Furthermore, we may supply in this projection a border scale for each parallel of latitude (see fig. 12), and in this way the scale variations may be accounted for when extreme accuracy becomes necessary.

Caution should be exercised, however, in the use of this projection, or any conic projection, in large areas of wide latitudes. The projection is not suited to this purpose. The extent to which this projection may be carried in longitude* is immaterial. It would be a better projection than the Mercator in the higher latitudes when charts have extended longitudes, and when the latter (Mercator) becomes objectionable. It can not, however, displace the latter for general sailing purposes, nor can it displace the Gnomonic (or central) projection in its application and use to navigation.

Thanks to the French, it has again, after a century and a quarter, been brought to prominent notice at the expense, perhaps, of other projections that are not conformal—projections that misrepresent

*A map on the Lambert Conformal Conic Projection of the North Atlantic Ocean, including the eastern part of the United States and the greater part of Europe, is now in preparation in the Coast and Geodetic Survey. The western limits are Duluth to New Orleans; the eastern limits, Bagdad to Cairo; extending from Greenland in the north to the West Indies in the south, scale 1:10,000,000. The selected standard parallels are 36° and 54° north latitude, both parallels being, therefore, true scale. The scale on parallel 41° (middle parallel) is but 1¼ per cent too small; beyond the standard parallels the scale is increasingly large. This map, on certain other well-known projections covering the same area, would have distortions and scale errors so great as to render their use inadmissable. It is not intended for navigational purposes, but is being constructed for the use of another department of the Government, and is designed to bring the two continents vis-à-vis in an approximately true relation and scale. The projection is based on the rigid formula of Lambert and covers a range of longitude on the middle parallel of 165 degrees.

forms when carried beyond certain limits. Unless these latter types possess other special advantages for a subject at hand, such as the polyconic projection which, besides its special properties, has certain tabular superiority and facilities for constructing field sheets, they will sooner or later fall into disuse.

On all recent French maps the name of the projection appears in the margin. This is excellent practice and should be followed at all times. As different projections have different distinctive properties, this feature is of no small value and may serve as a guide to an intelligible appreciation of the map.

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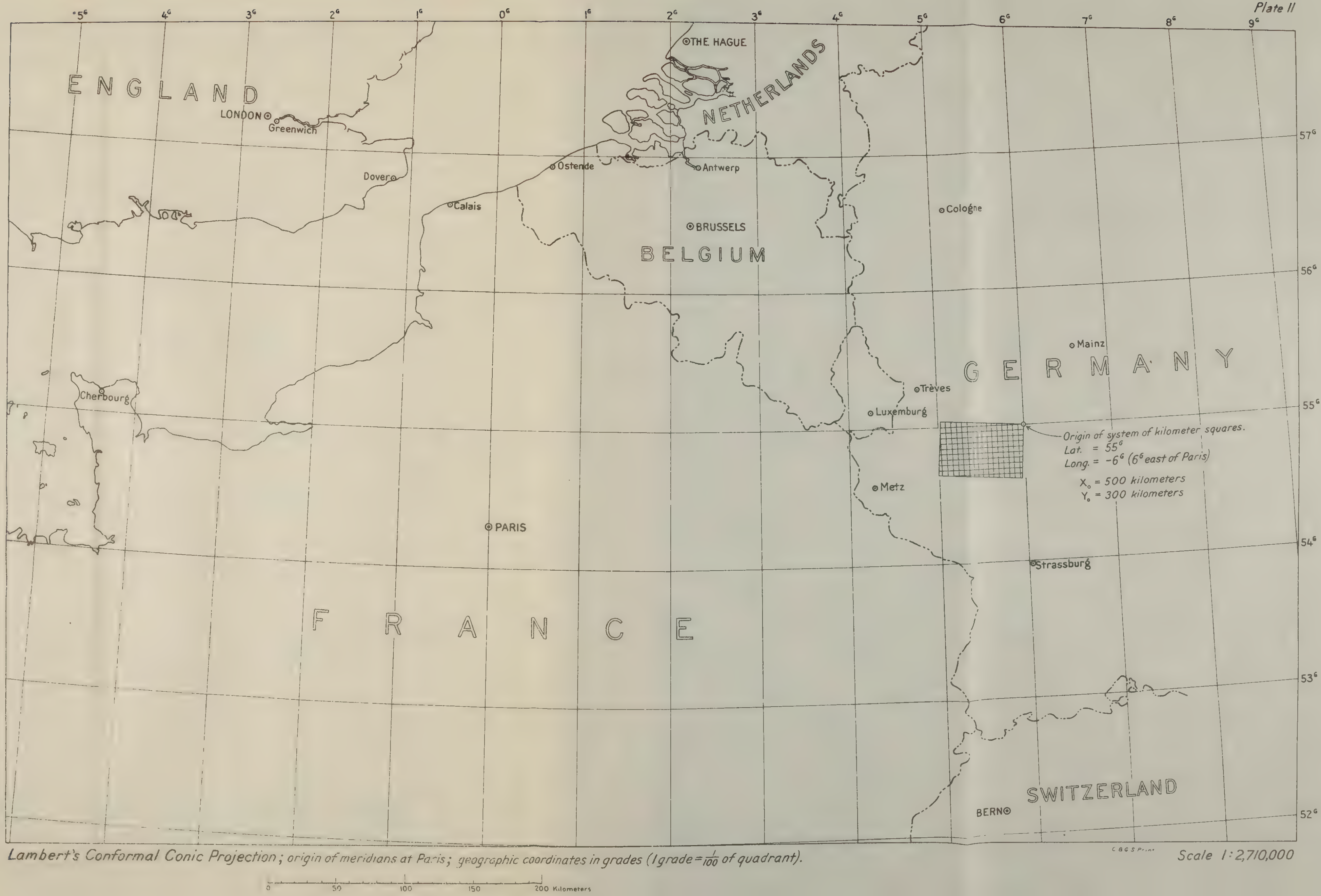
Lambert's Conformal Conic Projection; origin of meridians at Greenwich; geographic coordinates in degrees.

Scale 1: 2,710,000

Plate II

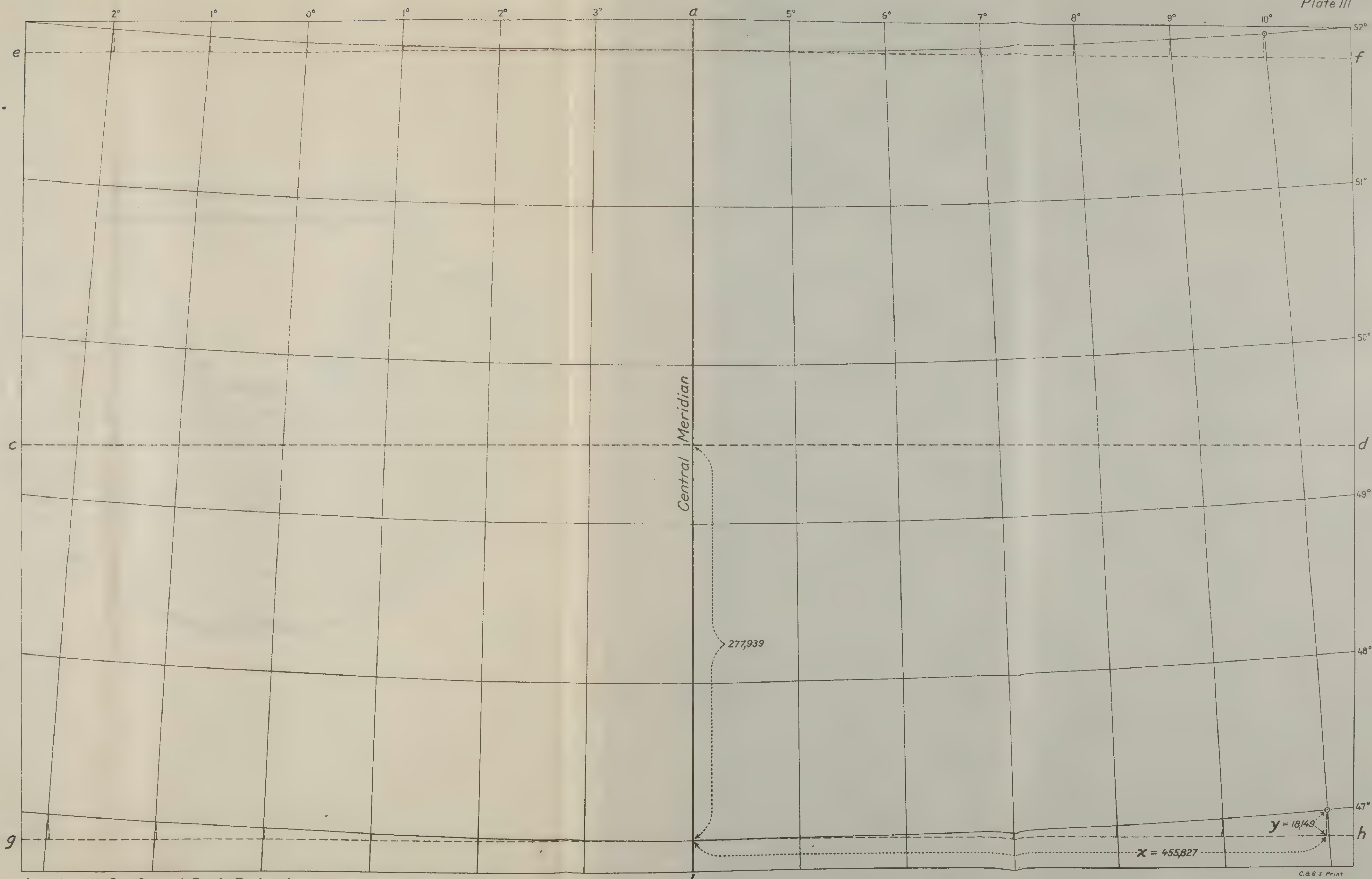


Lambert's Conformal Conic Projection; origin of meridians at Paris; geographic coordinates in grades (1 grade = $\frac{1}{100}$ of quadrant).

C.&G.S. Print.

Scale 1:2,710,000

Plate III

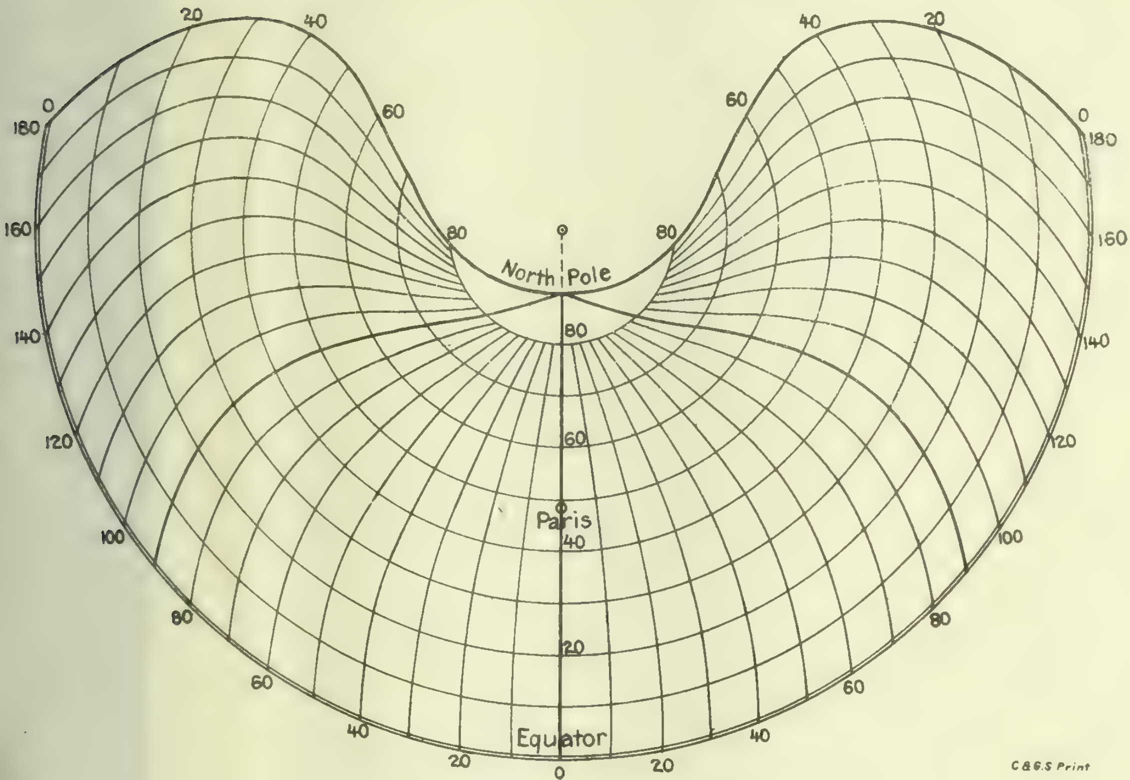


Lambert's Conformal Conic Projection — Construction Plate

Plate IV

BONNE PROJECTION OF HEMISPHERE

Development of cone tangent along parallel 45° N.



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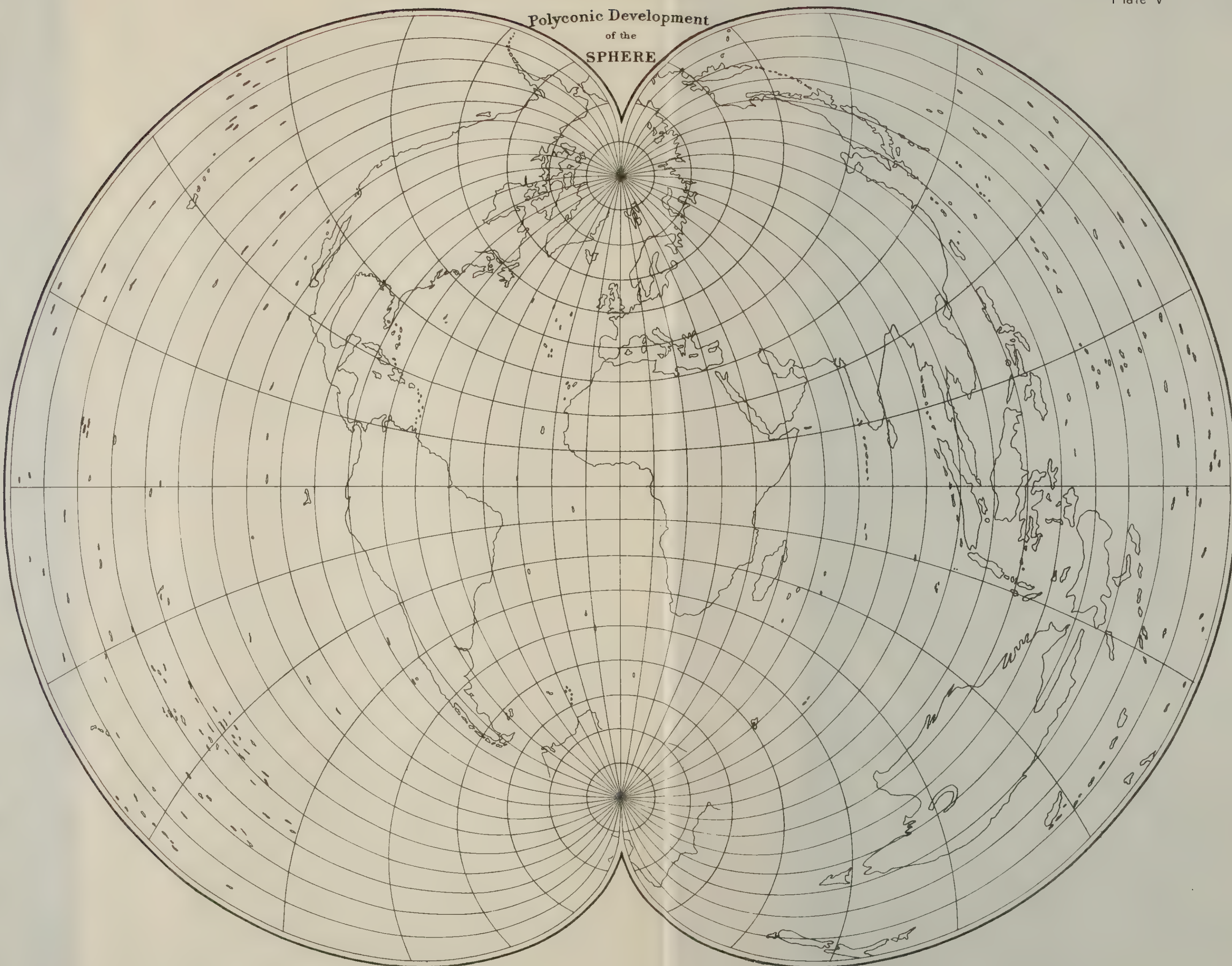
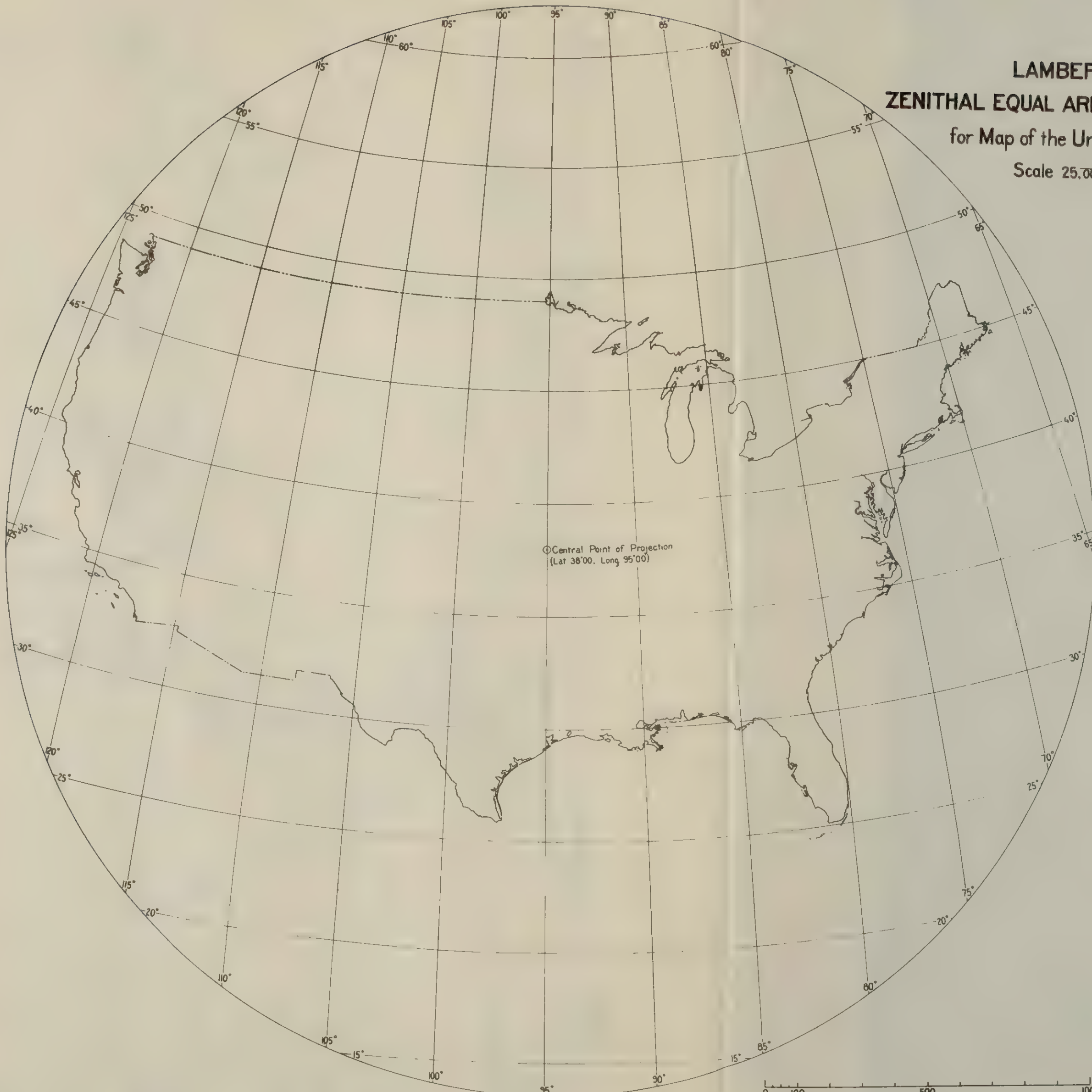


C&GS Print

LAMBERT'S ZENITHAL EQUAL AREA PROJECTION

for Map of the United States

Scale $\frac{1}{25,000,000}$



0 100 500 1000 Statute Miles

C. & G.S. Print

BAZANCOURT

Echelle de 1-20,000

Quadrillage kilomètrique Système Lambert